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NOVEMBER 1955

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A Journal for All Science and Mathematics Teachers

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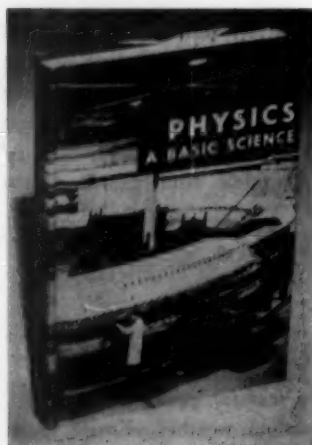
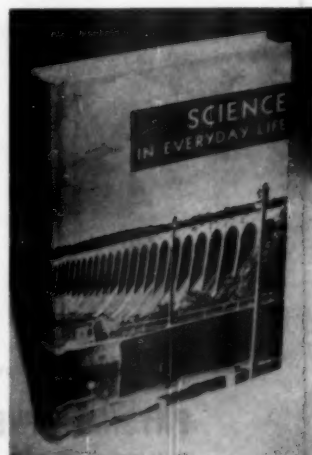
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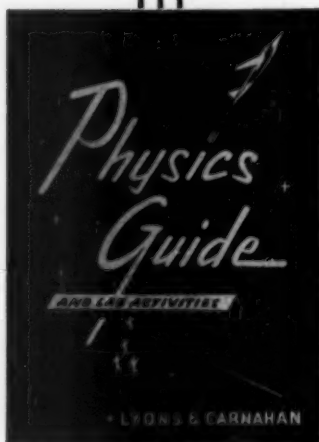
Wildly round our woodland quarters,
 Sad-voiced Autumn grieves;
 Thickly down these swelling waters,
 Float his fallen leaves.
 Through the tall and naked timber,
 Column-like and old,
 Gleam the sunsets of November,
 From their skies of gold.

—"The Lumberman."
 John Greenleaf Whittier

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SCHOOL SCIENCE AND MATHEMATICS

VOL. LV

NOVEMBER, 1955

WHOLE NO. 487

A CHALLENGE: SCIENCE FOR *ALL* RURAL YOUTH

JOHN H. CHILCOTT*

4090 Gary Street, Eugene, Oregon

Today is Tuesday: Lyle and Dan are out getting soil samples from the O-Bar-S Ranch . . . Mary is getting canned goods from the store for her experiment; Dick is already busy dissecting a fish . . . wonder how Dorothy is doing in the library—she is probably having difficulty finding information on the surface temperature of planets; Ron is still building the vibrograph down in the shop . . . Donna and Juanita are in the office giving each other a psychology test; Linda and Jacki are setting up their experiment in the fume hood. That takes care of everyone in Science II except Bob.

"Mr. _____, I'm ready for that trial test on mechanics."

"O.K., Bob, bring the slide rule up and let's see if you're good enough to use it on this test."

Just one year ago there was no organized science program in this small rural school (population 160), a school size which is by no means unique. There are about 200 schools of this size in the State of Washington and an even larger number in Oregon, Utah, Idaho, and Montana: about 1500 such schools in the Great Pacific Northwest—an area comprising almost one-fifth of the United States.

No science was being taught in the grade school. Science in the high school was dependent upon the capabilities of the teachers and the limitations imposed by lack of pupil interest and by the absence of suitable apparatus. Because there was a constant turnover of teachers, the boys and girls never knew what science, if any, might be offered from one year to the next. One might, therefore, find seniors taking general science or freshmen taking physics. These courses were of the

* Mr. Chilcott is a graduate student of the University of Oregon.

college preparatory type taught from a textbook yet the school was located at the foot of majestic Mt. Adams deep in the heart of the Pacific Northwest's snow capped Cascade Mountains, an area which is one of America's, if not the world's, greatest natural laboratories.

In this 100 square mile isolated valley the social and cultural activities were limited almost entirely to those furnished by the school: a band concert, a senior play, or a dance. Farm journals, county newspapers and mail order catalogues comprised the reading material of the majority of the natives. This was typical of the science dilemma to be found in an isolated rural school.



MT. ADAMS, WASHINGTON

In this day and age in which science has been made the tool of almost every layman, how can an inadequate program such as this possibly interest boys and girls in making use of science? Yet this was the challenge any science teacher accepted in this beautiful community where indifference reigned not only to science but to education generally. There were only two or three college educated people (foresters, not permanently settled) in the community, aside from the teachers.

The problem was to make science interesting and immediately useful. It was arbitrarily decided that these children would profit less from the study of the usual variety of detailed scientific processes than they would from a study of the role of science in serving their everyday needs and improving the standard of living in an agricultural and lumbering community.

The time for this interest in science to begin is when the child first enters school. Therefore, the first job was to encourage and to provide for grade school science. It was found that science fitted into the present program with a minimum of difficulty and a maximum of interest. The biggest obstacle was the feeling of inadequacy on the part of the teachers in the teaching of elementary school science. This was overcome by first gaining the teachers' interest in science, and then in helping them add to their knowledge through self study and the use of manuals which accompanied the various elementary science textbooks. These teachers were encouraged to make use of the more advanced material from the high school science library. Suitable materials for demonstrations and simple experiments were furnished by the high school laboratory. It was interesting to note that the following year the grade school children and teachers chose to spend their library money almost entirely on books related to science.

It was decided to teach general science in the eighth grade. This was to culminate the grade school science program and to introduce high school science. General science was taught by the high school science teacher who made use of the high school science facilities. Included in this terminal introductory course in general science was a brief history of the development of science, an explanation of the scientific method, and the study of the effect of science on present day living.

Another course called Science I, a course designed to meet the state requirement of one year of a laboratory science, was developed for grades nine and ten. This course included a unit which familiarized the student with laboratory materials and methods.¹ It also attempted to develop proper scientific attitudes. Within this unit the students studied the care and use of the microscope, micrometer, vernier calipers, bunsen burner, and scales; methods of measurement such as the metric system; laboratory safety and first aid. For each section of the unit a group of exercises was outlined. As each group of exercises was completed, the pupil took a trial test to determine whether he understood what he was doing. Finally, when all the trial tests were completed he took a final test on the whole unit. Before being permitted to continue on to the next unit he had to pass this cumulative examination. It is implied that each student could progress at his own rate under this system. This was valuable to the good students as well as to the poorer ones, since the better student could cover more material while the poorer student could realize some accomplishment without feeling left behind.

¹ A second unit, physiology and health, might be required if health was not taught in the physical education program.

Upon completion of the required work, the remainder of the year was spent in investigating various units. These units were selected by the student during a conference with the teacher. The choice of units ranged from college preparatory physics to gardening, and was based on the particular student's interests and abilities. For example, some students might elect to work on pre-college chemistry, while others could be working on units such as chemistry of the soil, chemistry in the home, or chemicals in industry. They were all taking chemistry but chemistry to meet their needs. In some cases units were built by the student and teacher to satisfy a specific interest.

Science II, although not a required course, was similar to Science I in that the students continued to choose units according to their interests. Science II included second, third and fourth year science students.

Both science I and II were offered each year, meeting in 100 minute periods three times weekly. These longer periods provided time for considerable laboratory activity as well as for supervised study. Most classes had only twelve students; making it possible to provide adequate individual attention. Individual discussion and explanation replaced more formal procedures. Some time was given to orientation before a student started a unit. Most pupils worked in pairs, but those whose interests did not coincide worked alone. Thus in a typical class period (Science II) one would find two students working on an experiment with an inclined plane (college-preparatory physics); two students doing an electrolysis experiment (college-preparatory chemistry); two students dissecting a fish (animal kingdom); one student measuring the liquid contents of canned goods (chemistry in the home); two students determining the acidity of a soil sample (chemistry of the soil); one student making a diagram of the different types of telescopes (astronomy); two students working in the health room (home nursing); and two students building a hot-bed (gardening).

At once the inadequacy of materials and references was apparent. At first some units could not be fully explored, but in many cases available materials were substituted and suitable apparatus was built and added to the laboratory equipment. So obvious was this inadequacy that the school board the following year purchased science furniture, laboratory apparatus, materials, texts, and source books. To provide for the greatest possible breadth of information for students and teachers, a wide sampling of textbooks was purchased in place of sets of textbooks. The library began to purchase current science publications. The students were encouraged to subscribe to science magazines through the school, at special low rates.

The advantages inherent in this program were many. Perhaps the most significant was provision for individual differences in interest and ability through the wide variety of science experiences offered and the methods of approach to science. The program was realistic, adjusted to the community, utilizing its natural laboratory and stimulating interest in science. Useful aspects of science were emphasized, rather than theory. The flexibility both from the administrator's standpoint and that of the students—plus the adaptability to inadequacies of materials and equipment—were also major advantages.

The chief problem was finding a teacher to head the program. A teacher for this type of program must have a broad science background, a willingness to learn with the students and the interest to work with grade school teachers. For Science I and II to be effective, the long class period was required. It is recognized that this could be an administrative disadvantage in some situations.

But it worked! From the time a student entered the first grade, he became science conscious, not as another "subject" in school but as a part of his everyday life. The high school students began to enjoy fulfilling the State Requirement of one year of a laboratory science. Those few preparing for college found that they could acquire adequate science preparation. Every single student was able to find that science really could become a part of his own interests and plans. There is no blueprint for a universal successful science program for it must be built around the needs, interests, and facilities of the particular school and community. However, since a program in a small school can be analyzed more easily than a program in a larger more complex school, this account may provide other teachers with ideas of how to solve the problem: of providing science which rural youth can use.

NEW COMET DISCOVERED NEAR POLE STAR

A faint comet has been discovered by an amateur astronomer in the constellation of Draco, the dragon, a stellar group that circles the North Pole.

The new comet was spotted by the Rev. Carl J. Renner of Castilia, Ohio, a member of the American Association of Variable Star Observers. Its magnitude is ten, too faint to be seen without a telescope.

The part of the constellation Draco in which the comet was found is nearly overhead and is close to Ursa Minor, the smaller bear. Both constellations can be seen circling Polaris, the Pole Star, throughout the year.

Its celestial position is 19 hours, 12.5 minutes in right ascension; 67 degrees, 33 minutes in declination.

Rev. Renner reported its motion as 5.6 minutes south southwest to Harvard College Observatory, clearing house for astronomical information in the Western Hemisphere.

THE PLACE OF GEOMETRIC CONSTRUCTIONS IN PLANE GEOMETRY

ADRIEN L. HESS

Montana State College, Bozeman, Mont.

INTRODUCTION

Many writers have insisted that straightedge and compasses constructions be given an important place in plane geometry. Other writers have been equally insistent that constructions with straightedge and compasses have no direct bearing on geometry as now taught or on the development of geometry in general, and that constructions are not an essential part of a proof (1; 2; 3; 4; 6; 7; 8; 10; 11; 12; 13; 14; 15).

In this study an attempt is made to determine the place of constructions in plane geometry. Fourteen recent plane geometry textbooks were examined to determine whether the trend in textbook content is the same as reported by Freeman (5) in 1932. More particularly, the proofs of the theorems were examined to determine what is the place of constructions in proofs of the theorems in these books. The exercises in twenty plane geometry achievement tests with copyright dates from 1925 to 1951 were examined to ascertain what use is made of straightedge and compasses constructions in these exercises. A questionnaire, containing selected questions and issues relative to straightedge and compasses constructions in plane geometry, was sent to 260 teachers who were members of The National Council of Teachers of Mathematics for 1952-53. The teachers were asked to check as many of the responses on the questionnaire as necessary to give their opinion on the question or issue and, if necessary, they were to add statements to further explain their opinion. Replies were received from 131 teachers and the responses were analyzed to determine the teachers' reactions to the questions and issues.

THE PLACE OF GEOMETRIC CONSTRUCTIONS IN FOURTEEN PLANE GEOMETRY TEXTBOOKS

A careful study of the fourteen books reveals that straightedge and compasses constructions occupy an important place in all these textbooks. Six of the books devote a chapter to constructions only, and seven of them devote a chapter to loci only. Two of the books include construction exercises in nine other chapters. In only one of the books is the use of tools other than the straightedge and compasses allowed for constructions. No reason is given in seven of these books for this restriction. In three of the remaining books the restriction is at-

tributed to Plato, in one it is stated that the restriction is imposed out of respect to the Greeks, in another it is stated that the theory of constructions is the chief concern and in two books constructions are based on postulates.

Of a total of 5748 pages in the fourteen books, 591 pages or 10.3 per cent are pages of constructions. The trend of a decrease, on a percentage basis, of constructions in plane geometry textbooks noted by Freeman in 1932 appears to be continuing in the more recent plane geometry textbooks.

Fifty theorems in which constructions are used in the proof and which appear in seven or more of the fourteen books were selected. The proofs of these theorems were studied to ascertain the use made of the constructions in the proofs. Constructions which make the visualization of a figure or relationship clearer are called "Visual Aids." Since Euclid postulated that:

- (1) A straight line may be drawn from any one point to any other point
- (2) A terminal straight line may be produced to any length in a straight line
- (3) A circle may be described from any center, at any distance from that center,

any such lines or circles are considered as existing whether drawn or not and are classified as "Visual Aids." When a construction is made in order that a definition may be used as a reason for a step in the proof, such a construction was classified as "Used as a Definition." When a construction is made in order that a postulate, corollary or theorem may be used as a reason for a step in a proof, it was classified as "Used as a Postulate, Corollary or Theorem." A construction which demonstrates that a proposed relationship exists was classified as an "Existence Theorem."

Out of a possible 700 items, the category "Used as Visual Aids" had a frequency of 484, the category "Used as a Definition" had a frequency of 22, the category "Used as an Existence Theorem" had a frequency of 22 and the category "Used as a Postulate, Corollary or Theorem" had a frequency of 18. Of these 546 frequencies almost 89 per cent of the uses is classified as "Visual Aids."

THE PLACE OF GEOMETRIC CONSTRUCTION IN TWENTY PLANE GEOMETRY ACHIEVEMENT TESTS

Of the twenty tests examined in this study sixteen are standardized tests. The total of the points on all the tests is 3109. If an exercise on a test was a construction for the student to do, it was classified as "Construction to Do." A construction given for the student to examine and then to designate the order of the steps in the construction or for the student to answer certain questions relating to the construction was classified as "Constructions to Examine." A question

or exercise relating to tools permitted in constructions was classified as "Questions on Tools." Any exercise or question that pertains to constructions but did not fall in the above categories was classified as "Questions referring to Constructions." Of the 3109 points on the twenty tests, 168 were "Constructions to Do," 112 were "Constructions to Examine," 34 were "Questions referring to Constructions" and 3 were "Questions on Tools." This is a total of 327 points or 10.5 per cent of the 3109 points. This result is larger than the 8.5 per cent reported by Pickett (9) in 1938.

RESPONSES OF 131 TEACHERS TO CERTAIN SELECTED QUESTIONS AND ISSUES

Of the 260 questionnaires sent out 131 or slightly more than 50 per cent were returned. Ninety-three teachers or 71 per cent wished to restrict constructions in plane geometry to those possible with straightedge and compasses. Regarding the reason for restricting constructions to those possible with these tools 68 teachers or almost 52 per cent were of the opinion that in geometry one is concerned with the theory of constructions and 72 teachers or 55 per cent were of the opinion that constructions are essential in the logical development of many proofs. Ninety-one teachers or almost 70 per cent wished to use constructions to familiarize students with terms, definitions, tools, etc., before formal proofs are introduced, and 77 teachers or almost 59 per cent wished to use constructions as an intuitive approach to geometry before formal proofs are introduced. When considering the possible purposes of constructions in the proofs of theorems in plane geometry 89 teachers or almost 69 per cent regarded constructions as visual aids in proofs and 85 teachers or almost 65 per cent considered constructions as an integral part of many logical proofs.

CONCLUSIONS ABOUT THE PLACE OF CONSTRUCTIONS IN PLANE GEOMETRY

The results of the examination of the fourteen geometry textbooks led to the conclusion that constructions occupy an important place in plane geometry. Constructions provide an excellent means of approach to demonstrative geometry and enable the student to become familiar with the concepts of geometry from a concrete basis. Constructions are included in several chapters in many of the books and this is approved by a majority of the teachers who responded to the questionnaire.

Constructions in plane geometry are restricted to those possible with a straightedge and compasses in thirteen of the books examined,

and a majority of the teachers who responded to the questionnaire concurred in this restriction. There is a considerable diversity of opinion among textbook writers and teachers as to why these restrictions should be maintained.

Although there is great emphasis on construction exercises in all of the geometry books studied, the percentage of construction exercises is less than what Freeman reported in 1932.

The study of twenty achievement tests indicates that there is at least as much emphasis on constructions in geometry tests as Pickett found in 1938. In some tests great emphasis is placed on the use of constructions, while in others the topic is omitted entirely.

From the results of the study of the use of constructions in geometric proofs in fourteen selected geometry textbooks it may be concluded that constructions have little use in the logical proofs of theorems in these books. This is in accord with the opinion of many writers, but is counter to the opinion expressed by a majority of the 131 teachers responding to the questionnaire.

IMPLICATIONS OF THE STUDY

There is an apparent disagreement among authors and among teachers regarding the place of constructions in plane geometry. The results of this study indicate that a selected group of teachers in the secondary school do not agree on the relationship of constructions to the proof of theorems. This suggests a needed emphasis in teacher education programs.

This study raises some questions which could be the subject of research by capable and interested secondary school teachers and by graduate students in mathematics education. Some of these questions are:

- (a) What change in the attitude of students toward plane geometry would result if construction work were reduced or eliminated?
- (b) What would be the advantages and disadvantages for the study of plane geometry if tools other than the straightedge and compasses were permitted in constructions, or if the tools of construction were further limited?
- (c) What changes need be made in present teacher education programs in order that teachers may become aware of the proper place of constructions in plane geometry?
- (d) If the questionnaire used in this study were sent to a number of teachers who are not members of the National Council of Teachers of Mathematics, would the responses to the questions on the questionnaire differ significantly from the responses of 131 teachers who were members of the Council?

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A GRANT FOR CANCER RESEARCH

A grant of \$100,000 for cancer research was accepted from the American Cancer Society by the University of Wisconsin regents.

Termed an institutional grant because it is given to the University for allocation, not to individual scientists for specific projects, the fund is to be used to make exploratory studies in promising fields and to open new vistas in cancer research.

Most grants given for University research are on a grant-in-aid basis. They are earmarked for a specific project by an established scientist.

Institutional grants are a new method of providing fluid research funds which can be used to follow new avenues of approach and make pioneer studies.

This is the fourth year an institutional grant for cancer work has been received by the University of Wisconsin from the American Cancer Society. The \$100,000 fund is the largest to date.

Some \$33,000 will be used by scientists in the University's McArdle Memorial Laboratory for Cancer Research. The remainder will be used by other University scientists in many different departments whose work has a bearing on the problems of cancerous growth.

Among projects to be supported are those concerned with the chemical composition of cells and cell particles, new methods of locating early cancers, trials of chemicals that might halt cancer, enzyme studies, studies of the origin of cancer in connection with protein and enzyme changes, chemicals that block cell growth, and analysis of nucleic acids that are cell constituents vital to cell growth.

Dr. Harold P. Rusch, director of the McArdle laboratory and chairman of the committee that allocates the American Cancer Society grant, points out that "institutional grants are large enough to enable scientists to explore new ideas and to do exploratory work in unknown areas that have not developed to an extent where the grant-in-aid type of support is justified."

SOME MIRROR TRIGONOMETRY

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To produce a "flat," three pieces are used: *A* is rubbed on *B* sliding and twisting this way and that, then *B* on *C*, then *C* on *A*, etc., etc.: when all three fit perfectly their surfaces are planes. When only two surfaces are rubbed in this way, the result is spherical. This is the way the amateur goes about making the concave mirror for a telescope. Some interesting trigonometry is connected with this process.

A spherical mirror has some disadvantages. What is called "spherical aberration" comes from the fact that a sphere has no true focus.

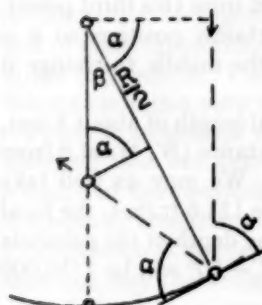


FIG. 1

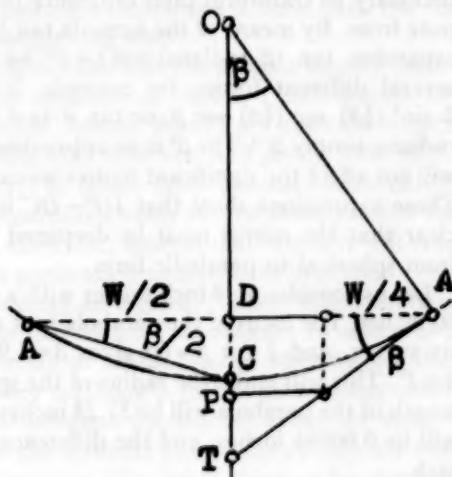


FIG. 2

A ray parallel to the axis of a spherical mirror (see figure 1) at a distance $R \cos \alpha$ from the axis is reflected through a point at a distance $(R/2) \sin \alpha$ from the sphere center. For very small β , this locates a point practically half way from the mirror to its center, and this point is called the focus of the mirror. However, if the width of the mirror is $2R \cos \alpha$, some of the rays strike the axis at a distance $\frac{1}{2}R(\csc \alpha - 1)$ beyond this "focus." If β is 2.6° , this distance amounts to only 0.1 per cent of the "focal distance," $R/2$.

But greater accuracy than this is desirable, and so after grinding the mirror down to a circular section, it is worked a little more to make the section a parabola, which has a true focus for parallel rays. To convert to the parabolic section, it is easier to do something in the

middle than to work around the edge. The curvature at the edge is retained: that is, the same tangent there is kept. Figure 2 shows how this is.

If the width of the mirror is $W=2(DA)$, the radius of the sphere will be $R=OA=(W/2)/\sin \beta$, and the depth of the spherical mirror will be $DC=(DA) \tan (\beta/2)=(W/2) \tan (\beta/2)$. The tangent to the circle at A is perpendicular to the radius OA, and cuts the axis at T, so $DT=(W/2) \tan \beta$. The parabola whose tangent is AT has its vertex at the mid-point of DT, at P: so the depth of the parabolic mirror is $DP=\frac{1}{2}DA \tan \beta=(W/4) \tan \beta$.

To show that P is deeper than C, examine $DP-DC=(W/4) \tan \beta-(W/2) \tan (\beta/2)=(W/4)(\tan \beta-2 \tan \frac{1}{2}\beta)$. For a small angle like β the tables give the same number for $\tan \beta$ and $2 \tan \frac{1}{2}\beta$, and it is necessary to transform their difference to get a formula we can compute from. By means of the formula $\tan \frac{1}{2}\beta=(1-\cos \beta)/(1+\cos \beta)$ the expansion $\tan (\beta \text{ radians})=\beta(1+\beta^2/3+2\beta^4/15+\dots)$, we can get several different forms: for example, $2 \cdot \tan \frac{1}{2}\beta \cdot \sin^2 (\frac{1}{2}\beta) \cdot \sec \beta$, or $2 \cdot \sin^3 (\frac{1}{2}\beta) \cdot \sec (\frac{1}{2}\beta) \cdot \sec \beta$, or $\tan \beta \cdot \tan^2 (\frac{1}{2}\beta)$, or if β is expressed in radians, simply $\beta^{3/4}$. The β^3 is an approximation, but the higher powers will not affect the significant figures we can get from this third power. These expressions show that $DP-DC$ is certainly positive, so it is clear that the mirror must be deepened at the middle to change it from spherical to parabolic form.

Let us consider an 8 inch mirror with a focal length of about 5 feet, 60 inches. The focus of the parabola is at a distance $(W/4) \cot \beta$ from its vertex, and $2 \cot \beta=60$ gives $\beta=1.909^\circ$. We may as well take $\beta=2^\circ$. This will make the radius of the sphere 114.6 inches, the focal length of the parabola will be 57.28 inches. The depth of the parabola will be 0.06984 inches, and the difference, $DC-DP$ will be 1/50,000 inch.

NAVY TO TEST LENS-LESS GOGGLES

The Navy, borrowing an Eskimo secret, will test goggles without lenses in the Antarctic this fall during Operation Deep Freeze.

Conventional goggles, use to protect the eyes from severe cold and wind, fog up rapidly and interfere with good vision.

The new goggles are transparent plastic-kidney-shaped cylinders fitted to a foam rubber frame. The cylinder circles the eyes and projects forward, shielding the eyes from glare. Although the cylinder is open at its front end, the still air within the cylinder deflects the wind. The goggles were developed from the Eskimo practice of cutting star shaped slits in goggles carved in whalebone. New cold weather clothing will also be tested in Operation Deep Freeze. The clothing is designed to keep a man afloat indefinitely.

The cold weather clothing has an outer waterproof shell, lined with perforated plastic foam that will not absorb water.

HOW TO CHOOSE A TEXTBOOK

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CRITERIA TO BE USED

Textbooks in general. One of the most important things a teacher needs to consider is the choice of a textbook. This is far more important than many teachers realize. Little care is shown in many schools in this respect and the students pay the penalty, because the choice of a textbook is something in which the students are vitally concerned. No particular textbook can be said to exhibit all the virtues, but there is coming to be a class of books, well constructed and modern in spirit, to which the intelligent teacher can turn if he has a fair set of criteria for making a choice and if, as is often not the case, he is permitted to choose his own text.

Because it is in the study of algebra that the student usually begins to appreciate a well-ordered sequence of topics, I will discuss the matter of choosing a textbook in algebra. What I say upon the criteria for judging a textbook in algebra, however, will apply with slight modifications to a geometry,¹ to books upon general mathematics, and to those texts which are prepared solely for the three years of the junior high school. I shall, therefore, assume that teachers will read this article with a view to its application to textbooks in general.

Points to be observed in choosing an algebra. No one would think of asserting that one person is just as good as another to select as a friend, and the comparison is not at all far-fetched when we say that it is equally true that not every textbook is just as good as any other.

The choice is not one that interests the teacher alone, for upon it rests not only the welfare of the students but, so far as they are concerned, the very existence of algebra. The list of points to be observed in the choice of a textbook, as set forth below, relates not to the merits of any special book but to a growing class of books which are modern in spirit and worthy in their construction. It is the result of the combined judgments of a large number of experienced teachers (who were my students) and may therefore be looked upon as at least fairly authoritative and representative.

General considerations. There are certain general considerations that enter into the choice of any textbook. These are:

1. *The purpose of the book.* Under the head of purpose the following questions arise:

a. Are the author's aim and particular point of view definitely set forth in the preface?

¹ See Mensenkamp, L. E. "Some Desirable Characteristics in a Modern Plane Geometry Text," Fifth Year-book, National Council of Teachers of Mathematics, *The Teaching of Geometry*, 1930, pp. 199-206.

b. If so, does the aim conform to the judgment of scholarly writers of the present time,—those who lead student teachers to impart a knowledge of sound mathematics in the most approved manner?

c. Does the book reveal clearly the purpose of the particular subject treated, thus giving to the student an appreciation of its importance in the lives of people in general?

d. Does the table of contents show a well-ordered sequence of topics, psychologically planned and so arranged as to show the significance of the work as a whole?

2. *The realization of the author's purpose.* To ascertain whether or not the author has realized his purpose, it is well to ask oneself these questions:

a. Is the author's aim, as set forth, realizable in practical teaching? An aim may be too high to allow of hitting the mark, or it may be too low, or it may be wholly lacking in accuracy.

b. Does the book reveal to the student the nature of the subject?—Does it make clear its importance, and its interesting features?

c. Is the book so constructed that it reveals the purpose of the subject in such a way as to give some assurance that the student will realize this purpose and will be inspired thereby to put forth his best efforts in his study of the subject?

3. *Subject matter.* It is exceedingly important that the nature of the subject matter be carefully scrutinized. This does not mean the mere counting of exercises or of problems,—a very crude way of reaching any conclusion whatsoever. It lies rather in considering the bases of selection of the material, and this is revealed by such questions as the following:

a. Are the recommendations of recent national studies and reports followed with respect to the vital topics to be considered? For example, with respect to algebra the report of the Joint Commission on "The Place of Mathematics in Modern Education" recommends the formula, equations, graphs, directed numbers, and numerical trigonometry as the topics of greatest importance.

b. Does the book emphasize these topics or does it contain merely the inherited material relating to operations or propositions that are no longer looked upon as essential?

c. Is the material related to the real needs of people? For example, are the formulas such as the student will need in reading books on elementary science, on the making of a radio set, or on ordinary business practice? In other words, is the material socialized as much as is reasonably possible?

d. Does the material give evidence of having been tested in the classroom, either by the author or under his direction?

e. Has the material been chosen after a careful consideration of the

relative value of the several topics? For example, has the author unduly emphasized the four operations with abstract expressions,—a line of work of very little value; or has he given relatively more attention to such an important topic as the interpretation of the meaning of graphs?

f. Are the main topics made interesting and full of meaning by the inclusion of helpful details? For example, has the chapter on numerical trigonometry been so presented as a part of algebra that the student sees it develop from the ancient method of *shadow reckoning*, and gains some slight idea of the way in which great distances are measured?

g. Using modern accepted objectives in the teaching of algebra as a guide, do you find that all obsolete and useless material has been omitted so far as present conditions allow? Such unimportant material includes traditional work that is not used in modern science and industry or in some part of advanced mathematics that is itself valuable.

h. Is the material so selected as to provide for a sufficient number of exercises to meet the needs of all types of students? In other words are the individual differences of students and of groups sufficiently provided for through an abundance of material of a kind that can be fully justified?

i. In the selection of material, is a proper balance maintained between the concrete and the abstract? For example, a book that has five hundred examples in the relatively unimportant subject of factoring, without a single genuine use of the topic, is wanting in balance. Similarly, a book that has four or five thousand abstract examples, with substantially no genuine applications, is hopeless. Mechanical efficiency has very slight value unless someone is going to use it somewhere and sometime.

j. Are the topics limited in the selection of material, so as to present the most important ones, treating these intensively?

k. Do the illustrative examples offer models which you would wish your students to follow in their written work?

4. *Arrangement of material.* After the appropriate material has been selected, there arises the necessity for arranging it to the best advantage for teaching. The following questions should therefore be considered:

a. Is the book made up of a series of unrelated chapters, seemingly without any connection; or is there a unifying principle that is increasingly apparent as the student proceeds? For example, is the dependence of one quantity upon another (the idea of functional relationship) made sufficiently prominent in the study of the formula, the graph, the equation, ratio, proportion, and variation?

b. Are the topics arranged according to a psychological sequence rather than merely a logical one?

c. Does the book follow the modern arrangement of topics, so that the most interesting and most frequently used ones come first and more abstract and less frequently used ones come later?

d. Is the material cumulative in arrangement, each part in general being used in what follows? This need not interfere with arranging certain minor details with a view to their omission without destroying the main sequence.

e. Is the material so arranged that each type of difficulty is introduced, on the principle of "one major difficulty at a time"?

f. Is each new topic followed immediately by carefully graded exercises, sufficient in number for the purpose of fixing the principle, but not so numerous as to become more routine material for keeping students busy?

g. Are the necessary definitions introduced when and only when the need for them is felt?

5. *Method of treatment.* Having considered the material and its arrangement, the method of treatment demands such attention as is suggested by the following questions:

a. Is provision made for the gradual learning of terms and processes, so that no avoidable congestion of difficulties shall occur at any one time?

b. Is the equation made subsidiary in importance to the formula? The student should see that the chief use of the equation is to enhance his ability to handle any kind of formula that he will meet with in science, in commercial mathematics, in measurement, or in advanced mathematics.

c. Does the book provide for a simpler treatment of certain traditional topics than was formerly the case? This question refers to such matters as the use of parentheses, simultaneous equations, the quadratic equation, factoring, fractions, and especially the four fundamental operations.

d. Is the checking of all work emphasized and thoroughly explained?

e. Is provision made, through correlation, generalization, and application, for transfer of abilities to other subjects and from topic to topic?

f. Do the explanations foster the ability to read the book understandingly?

g. Is the language of the book so simple as to be understood easily by the reader? Are the explanations stated in words and sentences that are readily comprehended?

h. Is the significance of the work rather than mechanical skill in unimportant operations comprehended?

i. Is the student led to see a worthy reason for each new topic? In pedagogical language, is the material well motivated?

Special features. Besides the general considerations that enter into the evaluation of a textbook, there are certain special features that demand attention. These are as follows:

1. *Applied problems.* The nature of applications demands the consideration of the following questions:

a. Are the applied problems as useful as possible, considering the student's limited knowledge of science, industry, and commerce?

b. Are these problems sufficiently numerous to emphasize the practical side of the subject under consideration, and is their importance in this regard duly emphasized?

c. Does the book recognize that problems may be genuine, in that they may be necessary in astronomy or in chemistry, and yet not seem real to the student at his present stage of advancement?

d. Does the book also recognize that a problem, say of the puzzle variety, may be very interesting to the student,² leading him to acquire the technique of solution, and still be not at all genuine and possibly be uninteresting to the teacher?

e. Are the problems clearly stated, so that there shall be no linguistic difficulties in addition to those of a mathematical nature?

f. Are the problems selected so as to afford plenty of variety and are they graded so that the student will approach the difficulties gradually?

g. Do the problems stimulate thought? For example, do they lead the student to consider whether more than one solution is possible, whether or not any solution is possible, and whether any change in statement leads to special solutions of particular interest? To take a simple illustration, the study of the family of problems included in the general case of finding two numbers whose sum is n offers an interesting range of possibilities such as the following:

(1) Suppose that the numbers are consecutive and $n = 9$.

(2) Suppose that they are consecutive numbers and $n = 12$.

(3) In each of the preceding cases, suppose that $n = 15$.

(4) Suppose that one number is $-2\frac{1}{2}$, or 0, or 11, or 17, and that $n = 15$.

(5) Suppose that $n = 0$ and that the numbers are both positive, are both negative, or are equal but have opposite signs.

h. Do the applied problems provide for the brilliant as well as for the dull student; for the rapid as well as for the slow student?

² Powell, J. H. *A Study of Problem Material in High School Algebra*, Bureau of Publications, Teachers College, Columbia University, New York, 1929.

2. *Reviews.* In no subject is a student expected to keep in mind all that he has learned. To provide for retaining the essentials it is necessary that a textbook should contain a reasonable number of carefully selected reviews. A satisfactory test of the book, with respect to this feature, is provided in the following list of questions:

- a. Is there sufficient number of reviews of the right kind?
- b. Do these reviews emphasize the fundamental parts of each unit of work?
- c. Are the reviews properly placed?
- d. Do the reviews encourage the student to organize his knowledge, selecting the important features, observing the connection of one topic with another, and building up the structure of the material as a whole instead of thinking of it as made up of detached parts?

e. To this end, are certain reviews cumulative, touching upon all preceding work and keeping the student refreshed as to all that has been studied before?

3. *Drill.* Besides the consideration of subject matter and reviews there is the equally important problem of drill work, not merely in algebra but in all branches of mathematics. This leads to the following questions:

a. Is there a sufficient amount of drill material and is it properly distributed in the textbook? No textbook can, within reasonable limits of space, provide for all needed drills and at the same time contain all the testing material necessary for maintaining the highest efficiency, but it can reasonably be expected to contain enough drill to prepare for such modern tests as are available for school use.

b. Is the purpose of each drill lesson made clear, so that the student may see the advantage of devoting his time of doing the work as well as he is able?

c. Is the drill material progressively graded so as to allow for the increase in difficulty in certain parts of algebra?

d. Does the drill work provide for individual differences among students, so that an assignment may increase or decrease in difficulty or in time allotment according to their needs?

e. In the interest of economy in learning, do the drill exercises cover systematically the various possibilities of error in the subject under consideration?

4. *Tests.* The rapid development of the testing movement has made it necessary for the schools to give careful attention to their construction. Tests are so important that the following questions may properly be considered in this connection:

a. Does each test meet the standard requirements of (1) validity, (2) reliability, (3) objectivity, and (4) comprehensiveness.

- b. Are the tests sufficiently numerous to be given with the necessary frequency?
- c. Are a reasonable number of the tests suitable for use as teaching devices as well as measuring instruments?
- d. Are the tests self-testing?
- e. Do they afford opportunities for making all possible errors under each topic, thus diagnosing the causes of the student's difficulties? In other words, are the tests diagnostic?
- f. Do they train in habits of speed as well as accuracy? That is, are some of them so devised as to be used as timed tests?

Aids in instruction. Not only is a textbook expected to supply subject matter, to arrange it psychologically, to select purposeful problems, to provide reviews and drills, and to prepare for modern tests, but it is expected to serve as an aid to the teacher in giving instruction, and to the student in receiving it. These two qualities should, therefore, demand our attention.

1. *As an aid to the student.* The following questions will assist the teacher in judging the textbook as an aid to the student:

- a. Does it begin by making the student feel at home in the subject and by setting him thinking about its significance?
- b. Does it give the student something to do as early as possible?
- c. Are the explanations brief, clear, and "to the point"? Are they such as the students themselves can readily amplify when they are discussed? The alternative is that they shall be tedious in style and discouraging in length, a shortcoming that should condemn any book.
- d. Does the book create an early appreciation of the value and beauties of the subject and a desire to pursue its study?
- e. Does it cultivate a desire for investigation, so that students will be encouraged to go further in the subject and in its applications to all branches of science and various phases of commerce and industry?
- f. Does the textbook lead the student to do some real, constructive thinking; or does it, on the contrary, give the impression that algebra is only a piece of mechanism and that geometry is a subject for mere memorizing?
- g. Does it aid in developing a reasonable attitude of mind toward problem solving, whether in algebra or any other branch to which it is related?
- h. Does it encourage the student to help himself rather than continually to depend upon the book or the teacher?
- i. In particular, does it give to the student plenty of opportunity for discovering by himself the principles of the science?
- j. Are suggestions given to the student for expressing his work in succinct form, and is neatness encouraged?

k. Do the illustrative examples offer models which you would wish your students to follow in their written work?

1. Do the illustrations aid in the understanding of the work? Are they valuable object lessons to the student for making their own drawings?

2. *As an aid to the teacher.* In considering the textbook as an aid to the teacher the following questions will be found useful:

a. Does the book contain helpful suggestions to the teacher and are they conveniently placed?

b. Is there a full and well-arranged index which permits of easily finding the special topics?

c. Is the table of contents sufficiently extended to show clearly the outline of the work?

d. Are all the necessary tables and other similar items of information given in the body of the text or at the end?

e. Is the number of applied problems, of abstract exercises, and of drill pages so extensive that the teacher will not, under ordinary circumstances, be called upon to supplement them any further than by a good set of modern tests?

f. Do the explanations furnish material for brief discussions in class as to the best way of proceeding in the solution of algebraic exercises and problems of all types?

The body, language, and soul of the book. It has often been observed that it is the intangibles which really count in our lives, and so it is with respect to a book. There are certain things that we can feel but cannot always measure. The following points have to do with qualities of this nature, some of which are easily measurable and others not.

1. *The body of the book.* As to the mechanical make-up of a textbook we are in a field in which we can, if we choose, apply a figurative yardstick, as suggested by the following questions:

a. Is the book generally attractive in external appearance? Is the binding pleasing in all respects?

b. Is each page a model of neatness? Is the material so arranged that the page is attractive?

c. Is the type clear, of the proper size, well spaced, and such as to avoid the danger of eyestrain?

d. Is the paper of good quality, strong, without undue gloss, and of a tint that is restful to the eye?

e. Do the illustrations fit properly into the general scheme? Are they clear and attractive? Are they in every respect the models that you would wish your students to copy?

f. Is the book a model of good craftsmanship, strongly built and artistic in every feature?

2. *The language of the book.* Less easily measured is the language of a book,—not the mere vocabulary or the grammatical constructions, but those intangible elements that go to make up the style which characterizes it. The following questions suggest the features to be considered:

a. Has the book such dignity of language and a style so concise and clear as to command the respect and appreciation of the students and to serve as a model in making their own work refined and attractive?

b. Are the discussions and the problems expressed in good English, the punctuation being correct and the phraseology suited to the students for whom the book is written?

c. Is the language characterized by simplicity so as to be clearly understood by all who read it, and is there no excessive burden placed upon the student by reason of a pedantic or unnecessarily labored and burdensome vocabulary?

3. *The soul of the book.* Still less easily measured is the spirit in which the book is written. The significance of this statement may be more clear if we consider the following questions:

a. Speaking figuratively, and allowably so, has the book a soul,—one that shines out so as to influence both the students and the teacher? Though intangible, does the spirit of the book lead the student to make friends with it and enjoy the time spent in its company?

b. Can you weigh the soul of a friend, or gauge it by points on a machine-made scale? Then why should we try to measure the soul of a book, or the soul of an author who writes the material for the pages and gives them the spirit which they figuratively possess? If the book has not this better nature, if it is nothing but mechanical drudgery, if it seeks to be judged only by the amount of obsolete work or by the number of problems it sets, then it has no spirit and is unworthy of a place in the schools.

"The faith we hold belongs not to us alone but to the free of all the world."

—DWIGHT D. EISENHOWER

More food and better food for more people. *No other nation in all history* ever had an agriculture so efficient that famine and hunger were merely conditions to read about under foreign date lines.

—JIN ROE in *Successful Farming*

Does the high school in your community offer courses in physics or chemistry, or are the children compelled to graduate without knowledge of these fundamental sciences?

MOTIVATING THE HIGH SCHOOL AND COLLEGE SCIENCE STUDENT

GERRIT VAN ZYL

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The teacher is by far the greatest motivating influence of the student. Teaching is enduring work. It is creative work. Teaching is a passion. Its motive is service to ambitious young men and women with open minds who are in need of guidance. To render such service to these impressionable young people, even in a small way, is the ambition and purpose of every good teacher. In order to be a good teacher, however, one must himself be motivated.

A number of conferences in biology, chemistry, mathematics and physics have been held for this very purpose. For high school teachers, especially, attendance at the annual meetings of the Central Association of Science and Mathematics Teachers, and other important professional secondary school science teaching groups, should be a yearly must. Facing, many times, over a hundred students a day for the school year can well become a tiring experience for the secondary school teacher. Experiences like CASMT meetings and regional conferences go a long way towards renewing a teacher's own enthusiasm for his work.

Other meetings sponsored by the National Science Foundation and the Division of Chemical Education of the American Chemical Society and a number of other scientific and educational organizations are all excellent professional morale boosters.

The teacher is the catalyst that inspires and activates the student. The catalyst may become "poisoned" or deactivated. The teacher needs to be rejuvenated by association with others having a common interest. He or she needs to brush up on new methods of teaching, new lecture demonstrations, new student laboratory techniques, and new discoveries presented by authorities in various fields. As a result of such experiences his renewed enthusiasm will be transmitted to students in his classes.

The classroom is the place for imparting such information, arousing curiosity and pointing out the opportunities offered by science; for clarification of fundamental theories and laws by free discussions between teacher and students, and for the demonstration of fundamental principles through sound student laboratory practice.

The student must be alerted to the importance and opportunities offered by science. Actually, the era of science has just begun. Robert E. Wilson, chairman of the board, Standard Oil Company (Indiana), visualizes the compression of the 500,000 years of man's development into fifty years, comparable to our own lifetime. He said:

"On the basis of this scale it took man 49 years to get over being a nomad. About six months ago a few men first learned to write; two weeks ago the first printing press was built. Only within the last four days have we really found out how to use electricity around which so much of our civilization is built. Within the very last day have come such amazing things as the radio, television, radar, diesel locomotives, rayon, nylon, sulfa drugs, penicillin, book-keeping machines, electric computers of inconceivably complex equations, 100 octane gasoline, color and sound motion pictures, and hundreds of other things which we take for granted. On our condensed time table, jet planes, dozens of new antibiotics and hormones, and the release of atomic energy all came into the picture this morning."

Examinations and tests are used to determine the extent to which the student has mastered the subject matter, but class discussions should be pointed toward developing the inductive and deductive reasoning power of the student. *The enthusiasm of the instructor for his subject largely determines the students' interest.*

Students should be allowed to express their opinions and differences of opinion and to cite results of independent laboratory investigation or references to support their opinions. The instructor should act as a moderator and have a positive answer whenever possible. Superior students can understand and study more advanced subject matter than is included in the textbooks. Teachers sometimes do not recognize the student's ability and willingness to learn new things. We should encourage the student to investigate any topic in which he shows an interest.

Lecture demonstrations are of great value, especially in beginning classes, in that they usually leave a lasting impression if they are properly chosen to illustrate a fundamental law or theory. When a student sees apparatus on the classroom demonstration desk he expects something new and interesting. He will try to think out the principles involved or be prompted to ask: "What is the principle involved? Does it have other applications? How does it work?" The more colorful or dramatic the demonstration the greater will be the interest aroused.

As an example, the new book, *Student Workbook for Lecture Demonstrations in General Chemistry* by Dr. Hubert N. Alyea, Princeton University, on laboratory demonstrations should be on every chemistry teacher's shelf. We have all heard the saying, "Curiosity killed the cat." Maybe so, but it has also led to an improved civilization and has stimulated the minds of students and teachers.

The well equipped, modern science laboratory is a place where talent may be discovered. It is the students' attitude towards learning by experiment, which counts most. The person who is bored by

experiments will soon lose interest in science. It should be impressed upon the learner that the prescribed experiments are not intended to be routine or cook-book methods, but that they are intended to provoke clear thinking and reasoning. Also, they are devised to illustrate principles which may have other applications. The laboratory is not a factory where its products are turned out by assembly line methods. No machine can stamp out qualities of leadership, imagination, ingenuity, enthusiasm or perseverance.

If the student shows a lack of interest he should be encouraged to



Dr. Gerrit Van Zyl, 1955 Scientific Apparatus Makers Association award winner in chemical education, looking over his prized scrapbook picturing the many degrees and honors won by his former students. With him is Eugene Heasley, graduating Hope senior, continuing his science studies on a scholarship at the University of Kansas.

Van Zyl, head of the chemistry department, Hope College, Holland, Michigan has been called the "epitome of good college teaching." His record of sending capable science students on to study for advanced degrees in science is practically unequaled among small colleges of the country.

devise an experiment which appeals to him. He should be allowed to proceed after he has the approval of the instructor. A student in chemical qualitative analysis, for example, may find his own sample, the content of which he is curious to know. The biology student may wish to demonstrate that "structure determines function" on various anatomical specimens not generally discussed in the textbook.

One of our college chemistry students, for an example, constructed his own apparatus for the measurement of dielectric constants at a cost of about sixty dollars to the chemistry department budget. Laboratory experimentation can be made to be exciting. *It is one of the teachers' most powerful tools for the motivation of the student.* It is a means by which one learns the scientists' logical thinking process. Brain-hand coordination is developed. The student learns to be neat and orderly and to be observant. He learns how to record experimental data in a systematic manner. Many of our best researchers are discovered in the laboratory. I am sure that every teacher hopes that he will be able to say as Sir Humphrey Davy said, "My best discovery was Michael Faraday."

Progress in science has been due in a large measure to the inquisitive experimenter—the research worker. While high school science departments many times do not have the time and the facilities for it, every undergraduate college should have, at least, a limited research program to motivate both teachers and students. The example of the teacher working in his laboratory or delving into the literature in the library incites his students to action. Small individual or group projects may be started as early as the freshman year on the college level.

It is not difficult to discern which students have research interest and ability. They may be invited to cooperate with their professor in his own research program. Many times outstanding high school science students have shown their worth as laboratory assistants to busy secondary school instructors, thereby easing a teaching load and at the same time providing a real challenge for the superior student.

By the end of his junior year the college student may be so stimulated that he will accept a scholarship or award for summer research. The work can then be continued in the senior year as a part of an advanced science course or as a course in special problems. If he is inclined to continue on a grant during the summer following graduation, he will probably become a co-author with his professor of an article in one of the scientific journals.

Much could be said about encouraging interest on the part of the student by other means. I will mention briefly only four. 1. The organization of science clubs such as the chemistry club, biology club,

pre-medical, mathematics club and physics club is highly desirable. Formation of these groups allow students with a common interest to become more closely associated. 2. Scholarships and prizes are an inducement to improved scholarship and increased interest. In our college, fourteen students received scholarships in chemistry ranging from one hundred and fifty dollars to five hundred dollars in 1954-1955. This scholarship program was made possible by the generous gifts of The Johnson Foundation, the Standard Oil Company of Indiana, the Du Pont Company and the Dow Chemical Company.

Prizes awarded to the most outstanding science student in each class are also good interest stimuli. (Parenthetically, these awards are also excellent grist for local publicity about the school's science program.)

3. The open house program for science students from neighboring high schools and citizens of the community at which a large number of demonstrations are in progress should be an annual event. 4. An annual science field trip to a large industry or to a research laboratory is also highly desirable. (Incidentally, your own previous visit to the field trip location and a discussion of the outstanding points to be observed on the student tour will greatly enhance the tour's worth for the student.)

True, some of the greatest early discoveries of science have been made with only meager equipment. However, today many "tools of science" are available. All of the manufacturers and dealers of apparatus and equipment and their technical representatives are willing and eager to cooperate with high schools, colleges and universities in supplying our student laboratories and demonstration preparation rooms at a reasonable cost.

On the college level the chemistry major, for example, must learn and understand the basic principles underlying the construction and operation of fractionating columns, the refractometer, polarimeter, polarograph, Ph meter, spectrophotometer and the cyclotron (a small unit is being built at Hope College). Otherwise the use of the instrument does not contribute to his education. Anyone can be taught to push buttons or to turn a dial and read a vernier.

The combination of all these things fosters the enthusiasm, ingenuity, initiative, good judgment and perseverance and the scientific attitude of the student, and prepares him for greater service to mankind. *But, the greatest motivating influence is still the good teacher.*

"We hold all continents and peoples in equal regard and honor."

—DWIGHT D. EISENHOWER

MAGNETISM

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CHAPTER VI

MAGNETIC POLES

In spite of the fact that we no longer consider magnetic poles as fundamental entities—as we do electric charges—there are still many interesting conclusions to be drawn from the calculated results obtained by manipulating magnetic poles as though they were isolated and fundamental entities.

Forces Due to a Small Magnet. It is common observation that one magnet affects another, or, as in Oersted's experiment, we notice that reactions occur between magnets and conductors in which electric currents are flowing. As already stated we shall want to emphasize more and more as we proceed that the real entities with which we are concerned in such work are electric charges. For the sake of understanding the older treatments it is proposed in this and the following chapter to consider some of the phenomena we call magnetostatics by the use of Coulomb's law for isolated magnetic poles. Later we shall consider these same problems from Ampere's point of view when we deal with the interaction of moving charges without the mechanism of any intervening magnetic field.

In Fig. 72, $N-S$ is a bar magnet whose distance between poles is l . We wish to know the force which it exerts at the point O , situated at a distance R from the center C of the magnet. The point O in Fig. 72 has been located at the origin of a system of rectangular co-ordinates while the magnet, $N-S$, is supposed to swing anywhere about the point O with its axis always parallel to the Y -axis. When this is done, θ is the angle between the radius vector R and the axis of the dipole, $N-S$, as well as the angle between the radius vector R and the axis of Y . At O will be placed a unit, positive, magnetic pole, ($m' = +1$), because the force exerted upon it due to the magnet, $N-S$, will be the so-called magnetic field intensity at O , and the direction in which the $+1$ pole will be urged will be the direction of that field. Both the intensity and the direction of the field are matters of definition. It will be assumed that this experiment will be carried out in a vacuum, so that the field intensity at a distance R from a single pole of strength m will be, by definition,

$$H = m/R^2,$$

which follows from Coulomb's law. In this problem the dipole, $N-S$,

* Deceased. Professor Williams died February 12, 1955.

will be treated as though it had two independent, isolated magnetic poles. Note that the N -pole can act on the unit magnetic pole placed at O and so can the S -pole. The field intensity of the N -pole at O is

$$H_N = N/R_2^2$$

while that of the S -pole is,

$$H_S = S/R_1^2$$

From experience it can be written that $N=S=m$ as the value of the pole strength of each pole of the dipole, $N-S$.

The total force acting on the unit magnetic pole at O will then be these two forces indicated above and represented in magnitude and direction by the two vectors, On and Os , in Fig. 72. As a matter of

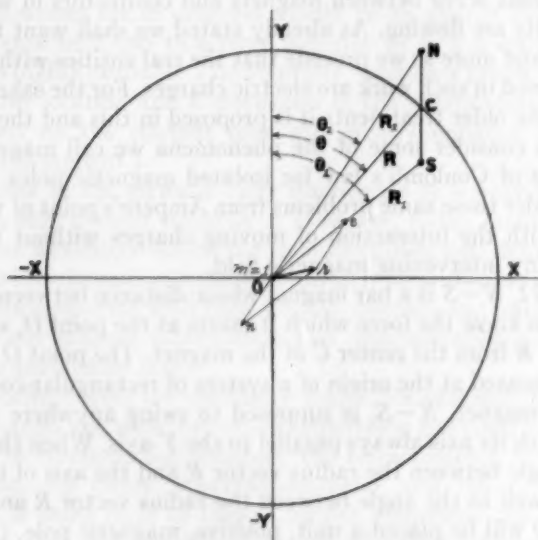


FIG. 72. The field of force about a small magnet, $N-S$.

convenience it will be found best to resolve these forces into their components along the axes of X and Y . When these are found, compound them into their resultant,

$$Or = H_R = \sqrt{F_x^2 + F_y^2},$$

the actual force with which the dipole, $N-S$, acts on a unit pole at the point O and represented in magnitude and direction by the vector Or .

For each pole, N and S , there will be two components, one along the X -axis and the other along the Y -axis, or there will be two components along each axis. Thus taking the X -axis first there will be two components along it, and the total force acting in that direction will be

$$F_x = m \sin \theta_1 / R_1^2 - m \sin \theta_2 / R_2^2$$

while for the Y -axis,

$$F_y = m \cos \theta_1 / R_1^2 - m \cos \theta_2 / R_2^2$$

It will now be recalled that in the original statement of the problem, the only terms given were θ , R and l . In these equations for F_x and F_y there are other values given, all of which are unknown. These are θ_1 , θ_2 , R_1 , and R_2 , equivalents for which must be found in terms of the known quantities. Write out all of these relationships and use what is needed.

Digression:

Assume that l in all of the following calculations is very small with respect to the value of R . It therefore follows that

$$R_1^2 = R^2 + l^2/4 - Rl \cos \theta = R^2 - Rl \cos \theta$$

$$R_2^2 = R^2 + l^2/4 + Rl \cos \theta = R^2 + Rl \cos \theta$$

$$R_1 + R_2 = 2R, \quad R_2 - R_1 = l \cos \theta, \quad \text{and} \quad R_1 R_2 = R^2$$

$$Y/R = \cos \theta, \quad \frac{Y - l/2}{R_1} = \cos \theta_1, \quad \text{and} \quad \frac{Y + l/2}{R_2} = \cos \theta_2$$

$$X/R = \sin \theta, \quad X/R_1 = \sin \theta_1, \quad \text{and} \quad X/R_2 = \sin \theta_2$$

$$Y = R \cos \theta \quad \text{and} \quad X = R \sin \theta$$

$$\sin \theta_1 - \sin \theta_2 = X/R_1 - X/R_2 = X(1/R_1 - 1/R_2) = \frac{R \sin \theta (R_2 - R_1)}{R^2}$$

$$= \frac{l \sin \theta \cos \theta}{R}$$

Similarly,

$$\cos \theta_2 - \cos \theta_1 = \frac{l \sin^2 \theta}{R}$$

Now returning to the original equations for F_x and F_y the foregoing values will be substituted wherever needed, and the equations can then be transformed as follows:

$$\begin{aligned}
 F_z &= \frac{mR_2^2 \sin \theta_1}{R_1^2 R_2^2} - \frac{mR_1^2 \sin \theta_2}{R_1^2 R_2^2} \\
 &= \frac{m}{R^4} (R_2^2 \sin \theta_1 - R_1^2 \sin \theta_2) \\
 &= \frac{m}{R^4} [(R^2 + Rl \cos \theta) \sin \theta_1 - (R^2 - Rl \cos \theta) \sin \theta_2] \\
 &= \frac{m}{R^4} [R^2(\sin \theta_1 - \sin \theta_2) + Rl \cos \theta (\sin \theta_1 + \sin \theta_2)] \\
 &= \frac{m}{R^4} \left[R^2 \frac{(l \sin \theta \cos \theta)}{R} + 2Rl \sin \theta \cos \theta \right] \\
 &= \frac{ml}{R^3} (\sin \theta \cos \theta + 2 \sin \theta \cos \theta) \\
 &= \frac{ml}{R^3} (3 \sin \theta \cos \theta) = M/R^3 (3 \sin \theta \cos \theta)
 \end{aligned}$$

where $M = ml$, the magnetic moment of the dipole.

Similarly,

$$\begin{aligned}
 F_y &= \frac{mR_2^2 \cos \theta_1}{R_1^2 R_2^2} - \frac{mR_1^2 \cos \theta_2}{R_1^2 R_2^2} \\
 &= \frac{m}{R^4} (R_2^2 \cos \theta_1 - R_1^2 \cos \theta_2) \\
 &= \frac{m}{R^4} [(R^2 + Rl \cos \theta) \cos \theta_1 - (R^2 - Rl \cos \theta) \cos \theta_2] \\
 &= \frac{m}{R^4} [R^2(\cos \theta_1 - \cos \theta_2) + Rl \cos \theta (\cos \theta_1 + \cos \theta_2)] \\
 &= \frac{m}{R^4} \left[R^2 \left(-\frac{l \sin^2 \theta}{R} \right) + 2Rl \cos^2 \theta \right] \\
 &= \frac{ml}{R^3} (-\sin^2 \theta + 2 \cos^2 \theta) \\
 &= \frac{M}{R^3} (\cos^2 \theta - 1 + 2 \cos^2 \theta) \\
 &= \frac{M}{R^3} (3 \cos^2 \theta - 1)
 \end{aligned}$$

As indicated above, the total force at O due to the dipole will be,

$$\begin{aligned}
 F_t = H_R = Or &= \sqrt{F_x^2 + F_y^2} \\
 &= \sqrt{\frac{M^2}{R^6} [(3 \sin \theta \cos \theta)^2 + (3 \cos^2 \theta - 1)^2]} \\
 &= \sqrt{\frac{M^2}{R^6} (9 \sin^2 \theta \cos^2 \theta + 9 \cos^4 \theta - 6 \cos^2 \theta + 1)} \\
 &= \sqrt{\frac{M^2}{R^6} [9 \cos^2 \theta (\sin^2 \theta + \cos^2 \theta) - 6 \cos^2 \theta + 1]} \\
 &= \sqrt{\frac{M^2}{R^6} (9 \cos^2 \theta - 6 \cos^2 \theta + 1)} \\
 &= \frac{M}{R^3} \sqrt{3 \cos^2 \theta + 1}
 \end{aligned}$$

In order to arrive at this quite simple but important result the assumption has to be made that l is quite small with respect to R . It is often necessary, in mathematics, in order to get any results at all, to make some simplifying assumption. If it had not been done in this case it would have required equations beyond our powers to handle with elementary mathematics.

Please note that in all cases the product $ml = M$ always appears. This is the magnetic moment of the dipole. Since it always does appear in these problems we could have started with it in the beginning and thereby have simplified the calculations greatly. This point will be enlarged upon later.

In the meantime, interpretation of the final results is quite in order, because mathematical analysis has no significance unless it can be interpreted. When $\theta = 90^\circ$, $\cos 90^\circ = 0$ and, therefore, the final result becomes,

$$F_t = \frac{M}{R^3} \sqrt{1} = M/R^3$$

This is for the condition that the magnet stands broadside to the point O . O lies on a line which is at right angles to the axis of the magnet and erected at the center of the magnet. This is known as the tangent (B) position used by Gauss in his method for finding the value of the earth's magnetic field. On the other hand, if $\theta = 0^\circ$, $\cos \theta = 1$, (A position of Gauss), then,

$$F_t = \frac{M}{R^3} \sqrt{3+1} = 2M/R^3$$

In this case the point O lies on a line which is a continuation of the

Significant was the technique for handling magnetic poles as such. There is also an economy in mathematics which it is well to observe if one is to be a good mathematician.

Fig. 73 shows a magnet, $N-S$, for which an expression is desired that will give the field intensity at the point O . This time, magnetic poles will not be employed directly, but magnetic moments will be introduced at once as indicated some pages back. $M=CA$ is a real entity. The radius vector R joining the center C of the magnet with the point O , makes an angle θ with the axis of the magnet. As a matter of convenience we may think of the actual magnet, $N-S$, as being replaced by two magnets, one parallel to the line CO and the other at right angles to it. If M is the magnetic moment of the original magnet then the two magnets replacing it will have magnetic moments represented by the expressions:

$$M_1 = M \cos \theta \text{ and } M_2 = M \sin \theta$$

M_1 is placed with its axis parallel to the line CO and the second magnet with its moment of M_2 is at right angles to the same line. Let it be emphasized again that the resultant effect of the two magnets will be the same as that due to the original magnet, for the field at O will be that compounded of H_1 , the field due to the first component magnet and H_2 due to the second. In doing this we place the two component magnets in the principal positions used by Gauss, called the "end-on" and the "broad-side on" positions. For the end-on component magnet we may write,

$$H_1 = 2M_1/R^3 = (2M \cos \theta)/R^3$$

which acts along the line CO and is represented by the vector OE , while

$$H_2 = M_2/R^3 = (M \sin \theta)/R^3$$

acts at right angles to CO and is represented by the vector ED . The resultant field, therefore, will be the vector,

$$\begin{aligned} OD = F_t = H_R &= \sqrt{H_1^2 + H_2^2} \quad (\text{Note } H_2 = H_1/2) \\ F_t &= \sqrt{\frac{4M^2 \cos^2 \theta}{R^6} + \frac{M^2 \sin^2 \theta}{R^6}} = \sqrt{\frac{M^2}{R^6} (4 \cos^2 \theta + \sin^2 \theta)} \\ &= \frac{M}{R^3} \sqrt{3 \cos^2 \theta + 1} \end{aligned}$$

Thus by a very simple process we have arrived at the value of the field found by a longer process in Method I.

Inasmuch as the components, H_1 and H_2 , which make up the total

force H , act along and at right angles to $CO=R$ we can see a definite relation between θ and ϕ , Fig. 73.

$$\tan \phi = H_2/H_1 = \frac{(M \sin \theta)/R^3}{(2M \cos \theta)/R^3} = \tan \theta/2$$

because by construction H_2 is drawn normal to CO at the point E in such a position that $CE=EO/2$. (Side-on effect of a magnet is $\frac{1}{2}$ of end-on effect.)

$$\alpha = \theta + \phi$$

$$\tan \alpha = \tan (\theta + \phi) = \frac{\tan \theta + \tan \phi}{1 - \tan \theta \tan \phi} = \frac{3 \tan \theta}{2 - \tan^2 \theta} = \frac{3 \sin \theta \cos \theta}{3 \cos^2 \theta - 1}$$

$$\cos \alpha = \frac{1}{\sqrt{1 + \tan^2 \alpha}} = \frac{3 \cos^2 \theta - 1}{\sqrt{3 \cos^2 \theta + 1}}$$

$$\sin \alpha = \cos \alpha \tan \alpha = \frac{3 \cos^2 \theta - 1}{\sqrt{3 \cos^2 \theta + 1}} \times \frac{3 \sin \theta \cos \theta}{3 \cos^2 \theta - 1} = \frac{3 \sin \theta \cos \theta}{\sqrt{3 \cos^2 \theta + 1}}$$

These values of $\sin \alpha$ and $\cos \alpha$ enable us to find the components of the force, F , along the X and Y axes and show that they check with the values found by the first method. Note that $OD=H$ may be resolved into H_1 and H_2 , or into H_x and H_y .

$$F_1 = H = \sqrt{H_1^2 + H_2^2} = \sqrt{H_x^2 + H_y^2}$$

$$H_x = H \cos \alpha \quad \text{and} \quad H_y = H \sin \alpha$$

$$= \frac{M}{R^3} \sqrt{3 \cos^2 \theta + 1} \times \frac{3 \cos^2 \theta - 1}{\sqrt{3 \cos^2 \theta + 1}}$$

$$= \frac{M}{R^3} (3 \cos^2 \theta - 1)$$

Also,

$$H_y = \frac{M}{R^3} \sqrt{3 \cos^2 \theta + 1} \times \frac{3 \sin \theta \cos \theta}{\sqrt{3 \cos^2 \theta + 1}}$$

$$= \frac{M}{R^3} (3 \sin \theta \cos \theta)$$

Whence,

$$H = \sqrt{\frac{M^2}{R^6} [(3 \cos^2 \theta - 1)^2 + (3 \sin \theta \cos \theta)^2]}$$

$$\begin{aligned}
 &= \sqrt{\frac{M^2}{R^6} [9 \cos^4 \theta - 6 \cos^2 \theta + 1 + 9 \sin^2 \theta \cos^2 \theta]} \\
 &= \frac{M}{R^3} \sqrt{9 \cos^2 \theta (\cos^2 \theta + \sin^2 \theta) - 6 \cos^2 \theta + 1} \\
 &= \frac{M}{R^3} \sqrt{3 \cos^2 \theta + 1}
 \end{aligned}$$

Once more we have arrived at the important result which gives us the value of the field strength about any magnetic dipole, whether a

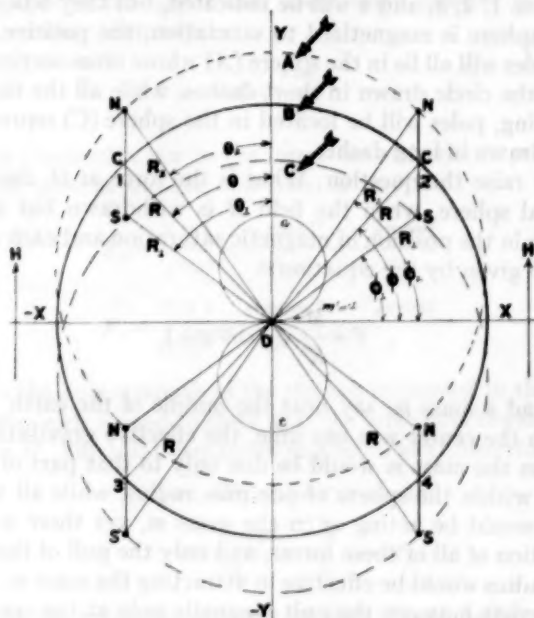


FIG. 74. A large sphere filled with innumerable but uniformly distributed dipoles.

magnet or a coil, provided the dimensions of the magnet or of the coil are small with respect to the distance R . The interpretation has already been given.

A Medium Filled with Magnetic Dipoles. Having found the expression for the magnetic field around a single magnet, or dipole, it is quite conceivable that by a process of integration it would be possible to find at any point within a medium filled with dipoles, and all turned in the same direction, the resultant force of all such fields. One may set up such an equation, but its solution is too complex for our purposes in dealing with simple mathematics as a tool.

By using some imagination, however, the problem can be solved quite readily. The following solution also illustrates that engineering and physics are rich in pictures, and all that is needed is to find the correct picture which can be expressed in simple mathematics.

In Fig. 74 is shown by a solid line circle the cross-section of a very large sphere, (*B*), of radius *R*, filled with magnetic dipoles turned in helter-skelter fashion. This group of dipoles has a magnetic field *H* applied to it in the direction shown. This turns all of the di-poles into the same direction as the applied field *H*. In this state the medium is said to be magnetically saturated. For simplicity's sake only four dipoles, Nos. 1, 2, 3, and 4 will be indicated, but they will show that when the sphere is magnetized to saturation, the positive, or north seeking, poles will all lie in the sphere (*A*) whose cross-section is represented by the circle drawn in short dashes, while all the negative, or south seeking, poles will be located in the sphere (*C*) represented by the circle drawn in long dashes.

We now raise the question: What is the force at *O*, the center of the original sphere, when the field *H* is withdrawn but all dipoles still remain in the position of magnetic saturation and each one exerts a field at *O* given by the equation:

$$F = \frac{M}{R^3} \sqrt{3 \cos^2 \theta + 1}$$

If one had a mass *m*, say near the middle of the earth where the distance to the center was one mile, the effective gravitational force acting upon the mass *m* would be due only to that part of the earth which lies within the sphere of one mile radius, while all the rest of the earth would be acting upon the mass *m*, yet there would be a neutralization of all of these forces, and only the pull of the sphere of one mile radius would be effective in attracting the mass *m*. A similar situation exists between the unit magnetic pole at the center of the original sphere and the attractions which the positive and negative poles in the two other spheres exert upon it. The unit magnetic pole at the center of the sphere (*B*) is off the centers of the other two spheres by a distance which is equal to half the length of the dipoles. Call this distance *l*/2. Consequently only the positive poles in the upper small sphere (*a*) will have any resultant effect on the unit magnetic pole and the same will be true for the negative poles found in the small sphere (*c*) below the center. The result of these forces on the pole is that the positive poles in the upper sphere (*a*) push *m'* = +1 downward toward the bottom of the page, and equally so the negative poles in the lower small sphere (*c*) attract *m'* = +1 downward toward the bottom of the page, and we see that the resultant

force at O is in a direction opposite to that which H originally imposed upon the point O . This is why we call this resultant force a *demagnetizing force*.

The radii of the two small spheres will both be equal to $l/2$, hence the volume of each one of the small spheres will be,

$$V = \frac{4\pi}{3} (l/2)^3 = \pi l^3/6$$

Next multiply this volume by the number of magnetic poles per unit volume ($=n$) and this will give the number of positive, as well as negative, poles in the upper and lower small spheres respectively. The total number then will be,

$$N = \frac{\pi n l^3}{6}$$

Now apply Coulomb's law to the forces between the N poles in each of the small spheres and the unit pole, $m' = +1$.

$$F_+ = \frac{N \times 1}{(l/2)^2} = \frac{-nm\pi l^3}{6l^2/4} = \frac{-2\pi nml}{3}$$

$$F_- = \frac{N \times 1}{(l/2)^2} = \frac{-nm\pi l^3}{6l^2/4} = \frac{-2\pi nml}{3}$$

where m is the pole strength of the dipoles imbedded in the medium. The negative sign is used to show that the force acts toward $-Y$ and opposed to H . The total demagnetizing force at O , therefore, will be,

$$H_d = (-F_+) + (-F_-) = -2F$$

$$= \frac{-4\pi nml}{3} = \frac{-4}{3} (\pi nM)$$

where M is the magnetic moment of each dipole being considered. Inasmuch as n is the number of dipoles per unit volume of the medium, then nM is the magnetic moment per unit volume. By definition the magnetic moment per unit volume is the same as the intensity of magnetization and, therefore,

$$H_d = \frac{-4\pi}{3}$$

$4\pi/3 = N$ is called the demagnetizing factor for a sphere. For other shaped bodies the value of N will depend upon the form of the body being magnetized.

This demagnetizing force may be visualized as in Fig. 75. Let H

represent a uniform magnetizing force which is applied to a piece of soft iron. As a result of the applied field, free magnetic poles, N and S , are set up in the iron. Each pole produces its own radial field, the resultant being the field due to a pair of poles. As will be seen from the direction of these lines of force between the poles, they are opposed to those representing H , the applied field. The effective field, H_{ef} , therefore, for magnetizing the piece of iron, if it is in the form of a sphere, will be,

$$H_{ef} = H - H_d = H - \frac{4\pi\mathfrak{J}}{3}$$

For a sphere H_d will be very large and as a result H_{ef} will be correspondingly small.

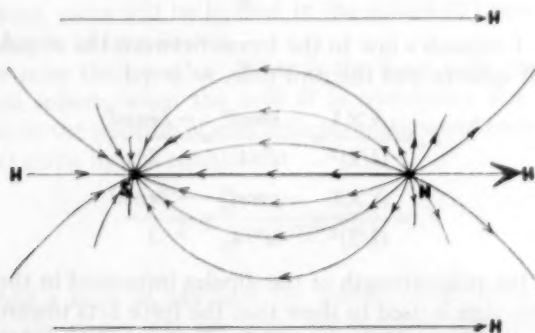


FIG. 75. The demagnetizing lines of force due to the poles set up when a ferromagnetic body is magnetized.

Demagnetizing Factor for Long Slim Rods.—The demagnetizing factor in a long slim rod of iron will be quite different from that of the sphere. The comparison between N for a sphere and N for a long slim rod of the same material will show what a difference the shape of the body makes in the value of the demagnetizing factor, N .

In the case of an iron cylinder, see Fig. 76, it may be thought of as made up of a series of alternate positive and negative poles which neutralize each other, except for the positive layer on one end and the negative layer on the other, when it is brought under the influence of a magnetizing force H . Again it will be seen, as in the case of the sphere, that the lines of force radiate out from the N -pole and into the S -pole, making the field in between them in a direction opposite to that of the applied field, H , and, therefore, constitutes a demagnetizing force, whose value may be obtained by calculation in the following way.

In Fig. 77 is shown a circular sheet of positive poles which represent the *N* end of the soft iron cylinder shown in Fig. 76, when it is magnetized by a field whose intensity is *H*. An equal force arises from the negative poles at the *S*-end. What will be the magnitude of the resultant of these forces on a unit positive pole at the center *C* of the cylinder?

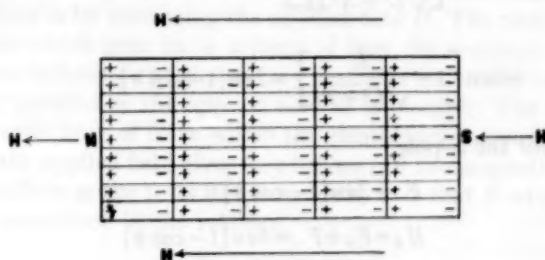


FIG. 76. Arrangement of dipoles in a long slim ferromagnetic body when magnetized.

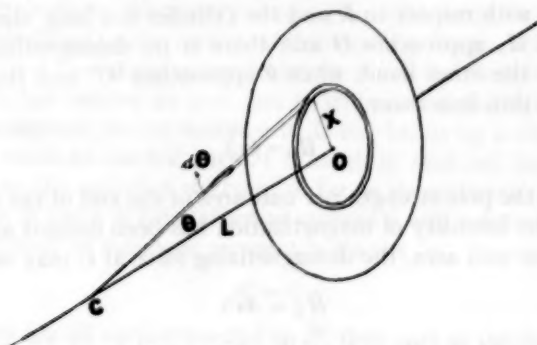


FIG. 77. The demagnetizing force due to a pole produced on a long, slim rod of ferromagnetic material.

Let $CO = L$ be the half length of the cylinder, whose radius is R , while n may be taken as the number of unit poles per unit area of the end of the cylinder. Consider the circular sheet of positive poles, or the end of the cylinder, as being made up of concentric circular strips dX units wide and take one of these strips at a distance X from the center O . The force dF on $m' = +1$ at C due to the poles included in this annular strip will be—parallel to CO ,

$$dF = \frac{1 \times ndA}{(\sqrt{L^2 + X^2})^2} \times \frac{L}{\sqrt{L^2 + X^2}} = \frac{2\pi nLXdX}{(\sqrt{L^2 + X^2})^3}$$

When summed up over the entire disk,

$$\begin{aligned}
 F_+ &= 2\pi nL \int_{x=0}^{x=R} \frac{XdX}{(\sqrt{L^2+X^2})^3} \\
 &= 2\pi nL \left[\frac{-1}{\sqrt{L^2+X^2}} \right]_{x=0}^{x=R} = 2\pi nL [1/L - 1/\sqrt{L^2+R^2}] \\
 &= 2\pi n \left[1 - \frac{L}{\sqrt{L^2+R^2}} \right] = 2\pi n [1 - \cos \theta]
 \end{aligned}$$

Similarly for the S-pole,

$$\begin{aligned}
 F_- &= 2\pi n [1 - \cos \theta] \\
 H_d &= F_+ + F_- = 4\pi n [1 - \cos \theta]
 \end{aligned}$$

This is the demagnetizing force at C when a ferromagnetic cylinder is magnetized longitudinally.

When $\cos \theta = L/\sqrt{L^2+R^2}$ approaches unity as a value, i.e., L is very large with respect to R and the cylinder is a long, slim piece of iron, then H_d approaches 0 and there is no demagnetizing effect, $N=0$. On the other hand, when θ approaches 90° and the cylinder becomes a thin iron sheet,

$$H_d = 4\pi 3$$

where n is the pole strength per unit area of the end of the rod. Inasmuch as the intensity of magnetization has been defined as the pole strength per unit area, the demagnetizing force at C may be written,

$$H_d = 4\pi 3$$

For the case of the thin sheet the demagnetizing factor is $N=4\pi$. These variations in N are all due to differences in shape of the bodies magnetized.

From the foregoing it will be seen that it is desirable in measuring the magnetic induction in ferromagnetic substances to use long, slim specimens, unless one wants to go to considerable trouble and correct for the demagnetizing factor.

Magnetic Induction. For many beginners the demagnetizing effect of the free poles induced in a piece of iron, for instance, when it is magnetized, has produced clouded thinking about magnetic induction. They go hand in hand, however, and it is urgent that a clear picture be made of what is happening in the process of magnetization. When a piece of iron, or other ferromagnetic body, is magnetized by bringing it into a magnetic field, the elementary magnets in the body

are eventually turned so that their axes align themselves in a direction as nearly parallel to the imposed field as possible. A perfect alignment is shown in Fig. 76. The magnetic field through each elementary magnet is parallel to the applied field, and they can't be made any more parallel. We say the iron has reached a *point of saturation*. The only way by which the total flux can be increased through the medium is by increasing the applied field H . The process of magnetization which goes on in a piece of iron, for instance, is nothing more than inducing the elementary magnets contained in it to turn as nearly parallel to the applied field as is possible. The addition of the magnetic lines of force which the elementary magnets possess to those of the applied field gives us what we call the magnetic induction of the medium as the total flux through it. It is that B which we have already mentioned in the relation

$$B = \mu H = H + 4\pi\mathfrak{I}.$$

and discussed somewhat in preceding chapters.

So far as we know there is nothing within the electric whirl we call a spinning electron (elementary magnet) to either increase or decrease the field produced by the motion of the rotating charge. Magnetically, it is simply a tiny circular conductor in which an electric current is flowing. It has neither an iron core to increase its magnetic moment, nor a diamagnetic one to decrease it. If one hangs up a simple coil of wire and sends an electric current through it, that coil has the same property as the spinning electron, so far as its magnetic property is concerned. It has a definite effective current C and a definite effective area A , so that its magnetic moment is always

$$M = CA$$

When they are all turned parallel to H , that part of the flux through the medium due to their turning cannot be increased.

Instead of the tiny magnets pivoted on needle points in Fig. 46, one could arrange to have a group of tiny coils with currents flowing in them so mounted as to rotate as did the magnets, and the results which Ewing found would have been repeated with the coils. To think of the electric whirls, found in the spinning electrons, (magnets) as just tiny air-cored coils of wire in which electric currents are flowing makes the understanding of *magnetic induction* much easier.

In Fig. 78 are shown two simple coils of wire, E_1 and E_2 . E_1 is stationary, and supplies the applied magnetic field H_1 . E_2 is arranged with moving contacts so that an electric current can flow through it, and yet be possible for the coil to turn about an axis in the plane of the coil and passing through its center. The electric current C_2 in E_2 will always be a constant. The directions of the fields in the two

coils are indicated by the arrows, $N-S$, and have values H_1 and H_2 respectively. The coil E_2 has a fixed and definite magnetic moment,

$$M_2 = C_2 A_2$$

When a current C is sent through the coil E_1 , it sets up a field H_1 to which the field H_2 in E_2 tends to set itself parallel. When this happens there is a maximum number of lines of force through the center O of the two coils. The space inside the two coils is as magnetically saturated as it can be with the currents employed. Let this total flux, i.e., the number of magnetic lines of force per unit area at the center of the two coils be designated as B , then

$$B = H_1 + H_2$$

(For simplicity's sake it is assumed that the fields H_1 and H_2 are uniformly distributed within their respective coils.)

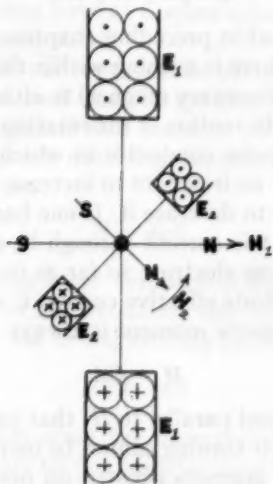


FIG. 78. The magnetic lines about two concentric coils.

Ampere pointed out that a magnetic shell could be represented by a certain current flowing along the periphery of the shell. The reverse representation may be made, and so for E_2 , if its magnetic shell equivalent is taken it can be looked at in the following way. If σ is the pole strength per unit area of the coil E_2 , then σA_2 = total pole strength of the coil whose magnetic moment $M_2 = C_2 A_2$. Already it has been shown that the number of lines of force emanating from a magnetic pole of strength m is $4\pi m$, so in this case,

$$m = \sigma A_2$$

and the number of lines of force issuing from the surface will be $4\pi\sigma A_2$. Furthermore, if l is the thickness of the magnetic shell replacing the coil then $(\sigma A_2)l = M_2$, the magnetic moment of the shell and lA_2 is its volume. Already the intensity of magnetization has been defined as the magnetic moment per unit volume, and so

$$\mathfrak{J} = \frac{l\sigma A^2}{lA_2} = \sigma$$

The intensity of magnetization has also been defined as the pole strength per unit area. Hence the lines of force emanating from unit area of the magnetic shell equivalent to the coil E_2 is,

$$4\pi\sigma = 4\pi\mathfrak{J} = H_2.$$

Inasmuch as B and H are also in terms of the number of lines of force per unit cross-section of the coil or of the shell, the equation,

$$B = H_1 + H_2 \text{ becomes } B = H_1 + 4\pi\mathfrak{J}$$

which is an exceedingly important equation when it comes to building dynamos, motors, transformers, and other types of electromagnetic machinery. From the way in which this equation was derived it must be evident that the term $4\pi\mathfrak{J}$ represents the number of lines of force added to H_1 by the turning of the electric whirls so that their magnetic axes coincide in direction with H_1 . When there are no electric whirls present then $B = H_1$.

This general equation for B has been derived from the consideration of two simple electric circuits. If in place of E_2 we put in a great many other small circular coils with currents in them, the argument would be exactly in the same and we would arrive at the same equation.

In deriving the equation,

$$B = H + 4\pi\mathfrak{J}$$

from a consideration of the magnetic fields of two simple coils in Fig. 78, nothing has been said about the demagnetizing factor N . The field H_2 must all go through the coils E_2 and E_1 in the direction indicated by the arrows. The return lines of force due to E_2 will in large part return outside of E_2 but inside of E_1 and a part outside of E_1 , because the field of E_2 theoretically extends to infinity. The lines of force from E_2 which return inside of E_1 act as a demagnetizing factor and H_2 is not the effective field, but

$$H_{ef} = H_2 - 4\pi\mathfrak{J}'$$

where \mathfrak{J}' is the intensity of magnetization due to the returning lines of force through H_2 .

$$\begin{aligned}
 B &= H_{ef} + 4\pi\mathfrak{J} = H_2 - 4\pi\mathfrak{J}' + 4\pi\mathfrak{J} \\
 &= H_2 + 4\pi(\mathfrak{J} - \mathfrak{J}')
 \end{aligned}$$

Inasmuch as \mathfrak{J} is dependent upon the number of electric whirls present per unit volume, there are not as many whirls in \mathfrak{J}' which get all of their lines of force back through E_1 as get them through in \mathfrak{J} in the opposite direction and parallel to H_1 . If these two quantities \mathfrak{J}' and \mathfrak{J} were equal then $B = H_2$.

It is this B which we call the magnetic induction of a substance, and it varies from H only as it has electric whirls associated with the space through which H operates. The greater the number of elementary electric whirls (spinning electrons) present for H to

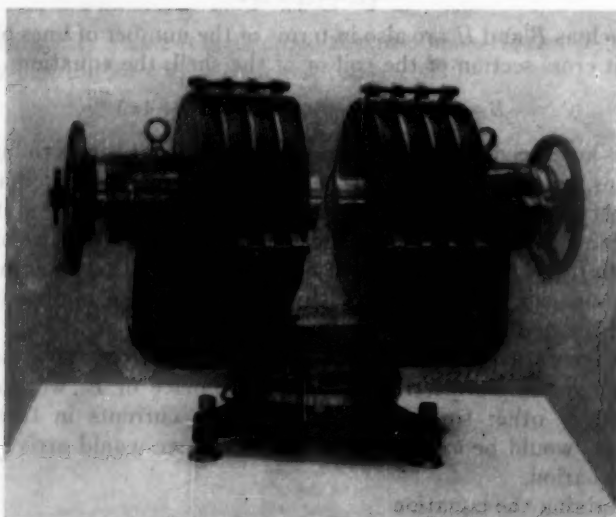


FIG. 79. An excellent form of electromagnet for producing powerful magnetic fields.

operate upon the greater will be the value of \mathfrak{J} and therefore of the magnetic flux B . We are limited at present in the values of H which we can attain. 30,000 oersteds in coils without iron cores have been secured for continuous service, while for electromagnets over 60,000 oersteds have been maintained. Kapitza obtained transient fields of about 320,000 oersteds. At present there doesn't seem to be very much hope in increasing the number of spinning electrons per unit volume in order to increase B .

If we divide the equation for B by H , i.e.,

$$B/H = H/H + 4\pi\mathfrak{J}/H$$

we get,

$$\mu = 1 + 4\pi k$$

where μ , the permeability factor, has its customary meaning, while k is called the susceptibility of the medium. The susceptibility of a substance tells us more specifically what the magnetic property of the substance is. It depends wholly upon the electric whirls present per unit volume. If no electric whirls exist then $\mathfrak{J}=0$ and $B=H$. A magnetic medium is determined by the presence of these elementary magnets whose magnetic fields are of the same quality as the applied field, so that B and H , therefore, are of the same quality and must have the same dimensions.

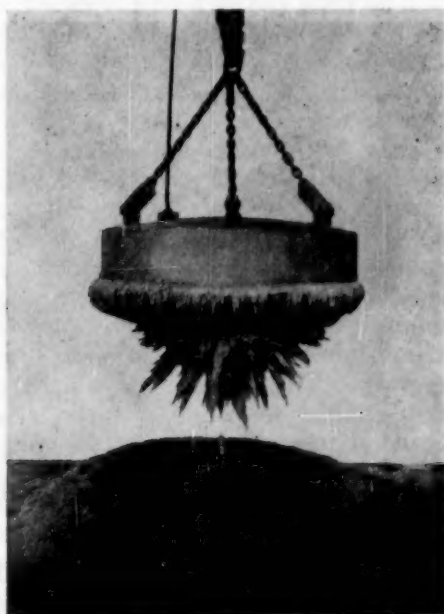


FIG. 80. An electromagnet for handling "pig" and scrap iron.

POWERFUL ELECTROMAGNETS

From the foregoing discussion it may be seen why ferromagnetic substances are used as the core of solenoids for producing intense magnetic fields. In Fig. 79 is shown an electromagnet for producing strong magnetic fields between the ends of the soft iron cores of the coils. As will be noted from an examination of Fig. 79 the iron core is almost continuous. The space between the ends of the poles is simply

a narrow slot in the magnetic circuit. The coils, as shown in Fig. 79, are wound from copper tubing through which cold water or some other coolant circulates. In this way large electric currents may be used for producing H without undue heating effects.

When the poles are brought quite close together the field between them is practically that given by B in the equation,

$$B = H + 4\pi\mathfrak{J}$$

Fields up to 45,000 to 50,000 oersteds may be maintained for some time by such an outfit.

Fig. 80 shows a powerful electromagnet used in handling scrap and pig iron. The electromagnet is lowered onto a pile of scrapiron,

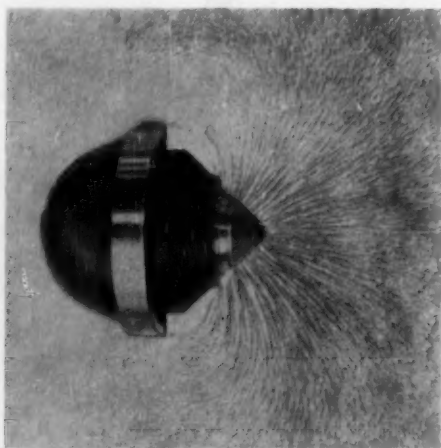


FIG. 81. A powerful electromagnet for removing iron particles from the eye.

an electric circuit (D.C.) is applied and a large load of the iron will adhere and be carried by the electromagnet to a point desired and dumped by simply breaking the electric circuit.

Fig. 81 shows an effective electromagnet for removing iron particles from the eye which often get into workmens' eyes in machine shops. The whole problem of efficient electromagnetic machinery—dynamoes, motors and transformers rests upon how large we can make B in the equation,

$$B = H + 4\pi\mathfrak{J}$$

Procedures for measuring B and particularly k will be discussed in succeeding chapters.

BROADENING YOUR SCIENCE EVALUATION PROGRAM

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Being "in the groove" means different things to different people. To the high school sophomore it probably indicates that a fellow classmate is "hep"; is really functioning smoothly along a certain line. "In the groove" may also be synonymous with "in a rut" which is where many teachers find themselves, especially in regard to evaluation practices. "Evaluation-wise," we are in a rut if we depend on the same testing devices to measure the same outcomes semester after semester. We are in a rut if we rely solely on true-false, multiple-choice, and matching questions to determine if John or Mary have achieved what we say they should from biology, general science, chemistry, and physics. Particularly in science does it seem easy to fall back on the "old reliables" in evaluation. Physical laws and principles haven't changed much over the years. The valence of hydrogen, carbon, and oxygen will remain the same in the foreseeable future. Fluids will continue to assume the shape of the container into which they are placed, and the amoeba will continue to pull himself about with his "false feet" for another million years. Because facts and minutia of science are in such abundance and present themselves so frequently, it is little wonder that testing is often confined to determining how well students have learned small details.

But regurgitation of the facts (be they important or not) is less than the minimum that we should desire from students. How students change their behavior and how the study of a subject improves the ability to think is really the important thing in learning. Our present testing systems are designed to reveal in only a very limited way, any change in behavior or ability to use concepts and principles. Some attention and teaching time should be devoted to the determination of how well boys and girls are able to use the science environment in which they are placed in school and in which they find themselves each day. We should try to discover the extent to which students are developing the ability to observe carefully and accurately the science happenings which transpire in and out of school. We should test to see whether boys and girls are improving their acuity in reading and interpreting data about nuclear energy, new cures for disease, and the new antibiotics about which they read daily in magazines and newspapers. Further, we should learn how keen is their evaluative judgment concerning the inferences they read in relation to the world of science. These outcomes are also worthy of an instructor's consideration and time in a modern science course.

DEFINING GOALS—FIRST STEP IN EVALUATION¹

Revamping a particular evaluation program requires considerable planning to make objectives and methods harmonize. As is often practiced now, testing is a matter of looking back over a unit to discover what items best lend themselves to the pencil and paper testing devices mentioned earlier. A modern evaluation program necessitates what might seem to be "putting the cart before the horse." Instead of looking backward at the end of a period of instruction, start at the beginning of a unit and define clearly what you expect the student to achieve. In addition to the usual subject matter, include some of the less often taught skills and attitudes. These are best realized when defined in simple language. For example, one might state, when preparing objectives for a unit, "At the end of this particular unit the students should be better able to apply principles offered in this unit to the following new situations." Or, "Using certain facts and data studied in the past six weeks, the class should have improved their skill in drawing conclusions and making generalizations in the following areas." Again, "using graphs, charts, or experiment sheets, the class should be able to interpret evidence and draw valid inferences about the following sets of data." It may not be possible to teach and test for all of these goals in any particular unit. It is better to choose one or two important outcomes and concentrate on these for a six weeks' period. Other goals may be stressed in succeeding units of study.

It is not enough to just have a list of objectives. The teacher must be sure these objectives are appropriate. To judge appropriateness, I would suggest application of the following key questions:

1. Can the pupils be stirred to desire these aims as their own? If not, learning will be imposed by the teacher. This may not be as harmful as many educators would lead us to believe, but it makes the task of instruction more difficult, and it seems almost certain that the majority of students will not continue to learn after the pressure is removed. Good motivation before the unit starts is the answer here.
2. Can the objectives be achieved with reasonable expenditure of time and effort? Do not be discouraged if the teaching and testing situation is awkward after the first attempt. Most first trials are. Streamlined methods will often develop as you become more skillful in administering the unit.
3. Are the chosen goals the most important ones? Facts, while im-

¹ The following steps were adapted from materials developed by Dr. Lucien B. Kinney, School of Education, Stanford University, Stanford, California.

portant, are not as vital as the ability to draw conclusions, make inferences, and to think critically in terms of the subject matter.

STEP TWO—DEFINING STUDENT BEHAVIOR

The second step in planning a modern evaluation program is to decide how a student acts when he has achieved the desired goals. You can be sure your teaching program is effective in the area of developing the ability to make generalizations when the student volunteers reasonable generalizations during his discussions in class or otherwise. A boy or girl is learning to interpret data when charts or graphs from recent papers or periodicals are presented and the student realizes the limitations of the data or sees clearly the implications of what he has read or been told.

STEP THREE—FINDING USEFUL TESTING TECHNIQUES

Finally, once objectives have been set and behaviors defined, the job of the teacher is to find appropriate techniques for obtaining and recording information. This is usually the most difficult task of all. First, it requires that the instructor be willing to experiment and to agree beforehand that first attempts, even though seemingly not as perfect or as reliable as more "tried and true" methods, can be revised and reused to better advantage. Second, it means that the testing devices will have to be developed over a period of time, and will require frank sessions of critique between the instructor and students. In fact, the students can well be told that new testing devices are being used, and that their cooperation and honest criticism will be appreciated.

There are several areas in which evaluation programs may be broadened. The remainder of this article will take up four of these and illustrate the use of each.

1. *Evaluating for speed and understanding in reading science materials*

When teachers try to analyze why a student is a poor or good learner, they soon discover that the student's ability to read is an important factor. This is perhaps even more true in science where the vocabulary takes on specific meanings. It is interesting to note that high scores on standard reading tests do not always indicate a similarly high ability to read science materials. This may be due to newness of words or because the student tries to read science literature in the same fashion he does fiction.

The following material can be considered as a combined learning and testing device if it is administered properly. Incidentally, any test worth its "salt" should afford the children a chance to learn,

both through the way the test is organized and by thorough review of the test when the papers are returned. The general form for the test described below has been worked out by C. R. Nelson² for general science, but the pattern may be adapted to any science. The test is constructed to give two scores; one based on the combination of speed of reading in relation to accuracy, and the second, accuracy alone.

The reading material consists of approximately one and one-half double-spaced pages of true but unfamiliar reading excerpts about a certain subject. We are not concerned with what the pupil knows; only with what he can get from taking the test. The teacher makes the first statements simple and factual, but the later statements increase in complexity of content and structure. The reading material is followed by a test of about 15 questions, the answers to which may be marked true, false, or doubtful.

The pupils are instructed concerning the following items in the administration of the test. When the students begin the test, the teacher will begin to time their progress. At the appropriate time after the test has begun, the teacher will announce "Four minutes," and will thereafter indicate ten second and one minute intervals on the board. The pupils will record the number of minutes and seconds required to *read and complete the test*. The pupils should not refer to the test before reading the material, but they may refer back to the material at any time while taking the test. They should be told to consider only the reading material as a source for answering questions.

When the last pupil announces his time, papers are exchanged and students score the papers. "Communal grading" provides discussion which enables students to see what kinds of errors they made and why they made them. Each student then makes the following computations. One, they compute the time in seconds that it took to complete the test. Second, they divide the number of seconds by the number of correct answers. This last number will provide the number of seconds to get one correct answer.

The teacher collects the papers and tabulates the scores in the following fashion. (1) Arrange the papers in order from highest to lowest according to the number of seconds to get one correct answer. One-fourth of the pupils, those having the lowest scores, are called reading group, group IV, the slowest. (2) A distribution is also made of the number of test items correct. Each group is labeled 1, 2, 3, 4, with *arabic* numerals.

The teacher records both groupings and returns the test to the child. A typical designation may be 1/2 or IV/3. The roman numeral

² For further information, write to C. R. Nelson, Chairman, Science Dept., Weeks Junior High School, Newton Center, Massachusetts.

indicates the group on the basis of combined accuracy and speed, while the arabic number indicates the group on the basis of accuracy alone. The teacher then points out (if the students already haven't noticed) that a student may be classed as a:

1. Fast, accurate reader.
2. Fast, but inaccurate reader.
3. Slow, accurate reader.
4. Slow and inaccurate reader.

Group 1 is told to increase their reading speed without loss of accuracy. Group 2 is told to slow down a bit to improve on accuracy. Group 3 should find ways to increase speed, and group 4 is told to be more concerned with accurate reading first, then speed. Pupils may be referred to special consultants or to the English or speech teacher for help. The science teacher may want to give direct help where possible by noting and correcting such simple items as lip movements, a large number of eye fixations per line, or regressions.

This testing device should be repeated three or four times a semester during which time it is stimulating for each student to chart his progress.

2. Evaluating pupil's resourcefulness in the use of original apparatus

Much of our science in the laboratory is taught "cook-book" fashion. The students are told to add so much of "A" to a certain amount of "B" to obtain "C." To achieve this astounding bit of science skill the student is further instructed how to assemble the apparatus, being helped at all stages by diagrams and pictures. There is no particular case *against* helpful diagrams, but so often the students leave a laboratory situation unable to apply the principles or the apparatus in any fashion other than that prescribed in the manual.

To test how fully students understand experimental apparatus and to test and develop their ability to apply familiar equipment in unfamiliar situations, the following testing situations are suggested.

Groups of equipment similar or identical to that used in class are placed on desks or wall benches about the room. Near each group is placed a 5"X7" card on which are the directions or requirements for the use of the equipment. When a test is given, students are rotated from desk to desk and are given one to two minutes on each question, or they are permitted to go the wall benches at their leisure during the testing period. In the latter method each student is permitted as much time as he wishes at each "station," but no more than one student is permitted at a "station" at a time. This method has been used successfully with 28 students, using six stations during a testing period of 50 minutes. These items did not comprise the entire test;

there were also questions of the more traditional type which the students answered at their seats. This technique should be introduced to the class before using it on an examination. It is recommended that one or two questions be prepared and used in place of a ten minute written quiz. Some illustrative situations which might be used appear below.

FOR CHEMISTRY

1. GIVEN: Bunsen burner, fastened down; gas supply; matches; short rubber tubes; glass tubes.
REQUIRED: To light the Bunsen burner without moving it.
2. GIVEN: A mixture of sugar, sand, and iron filings; some water; magnet; towels.
REQUIRED: To describe how to separate each of the three substances in the mixture from the other two.

FOR GENERAL SCIENCE OR PHYSICS

1. GIVEN: Two bar magnets, one with the N and S poles identified, and the other with one blue and one red tip only.
REQUIRED: To find whether the red or blue end is the N pole.
 2. GIVEN: Two bottles of odd shape, nearly the same in size; pan of water.
REQUIRED: To find which bottle holds more.
 3. GIVEN: A light bulb with the wattage marked on the end.
REQUIRED: To tell how many amps the lamp draws on an ordinary house circuit. (The solution depends partly on the student being observant enough to pick up the bulb to discover the proper wattage, and to then apply the general formula: $\text{Watts} = \text{volts} \times \text{amperes}$.)
3. *Evaluating ability to interpret data from experiments and data found on charts and graphs*

A brief glance at any common periodical or the daily newspaper will disclose how much information is dispensed through charts and graphs. On the basis of these statistics, authors draw inferences and ask that the reader subscribe to an opinion or product on the basis of these inferences. To help the student better understand the propaganda to which he is subjected, it seems wise to help him interpret intelligently what he reads in charts and graphs. In presenting this type of evaluating device, the following statement should be printed for the student to read.

Part 1. Inferences.

Directions: In the following exercise an experiment or situation is described. Below the description of the experiment are several statements concerning the experiment.

- Mark with an: (X)—every statement which you believe is a reasonable statement in the light of the experiment.
(0)—every statement which cannot be true because it is contradicted by the results of the experiment.
(-)—every statement which *might* be true or false because you do not have enough information given in the experiment.

Base your decisions on the results of the experiment plus any scientific information you have obtained from studying in class.

PROBLEM #1. Some students collected dead leaves and pond water which they brought back to the classroom and placed in a series of covered quart jars. The jars were divided into four groups and filled as described below. The water was then examined with a microscope every day for a period of two weeks to see if living protozoa were present. Assume the results in the groups listed below are not the result of contamination.

The Jars Contained:

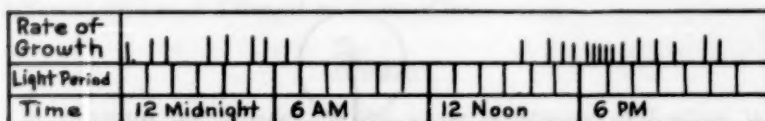
Results

Group 1—Pond water and dead leaves	Protozoa present
Group 2—Tap water and dead leaves	Protozoa present
Group 3—Pond water only	Protozoa present
Group 4—Tap water only	Protozoa absent

Mark the following conclusions as instructed above:

- _____ a. The protozoa seen in Group 1 may have come either from the dead leaves or from the pond water.
- _____ b. The protozoa seen in Group 2 came from the dead leaves and not the tap water.
- _____ c. In Group 1, a greater variety of protozoa came from the leaves than from the pond water.
- _____ d. Protozoa must have been on the dead leaves.
- _____ e. Living protozoa probably would have been found if boiled leaves had been placed in boiled pond water.

PROBLEM #2. Below is a record of the growth in height of a tobacco plant during a continuous 24 hour period. Each vertical mark in the upper row of the diagram represents a growth in length of stem of one-three-hundredths of an inch.



Mark the following conclusions as instructed above:

- _____ a. The plant grew more rapidly during the hours of darkness.
- _____ b. The intensity of light on the plant was greater between the hours of 7 A.M. to 8 A.M. than between the hours of 4 P.M. to 5 P.M.
- _____ c. The plant grew most rapidly just after 6 P.M. and just before 6 A.M.
- _____ d. The plant grew most rapidly during the three hours just after midnight.
- _____ e. The rate of transpiration was greatest after 6 P.M., therefore the plant had more water to use in growth.

The problems listed above utilize principles which might normally be a part of the academic training in a biology course. To help the student transfer this training to other areas of living, and after he has mastered this form of test, it is but a natural step to devise problems taken from the daily papers or periodicals. Sample problems might concern:

1. Experiments describing the effects of tobacco tars on white rats
2. Charts showing the results of mass polio vaccination.

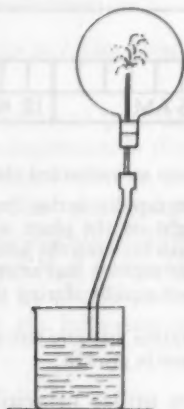
3. Pseudo-experiments used to substantiate advertisement in everyday literature.

4. *Evaluating the ability to observe*

It is often discouraging to note that many students make mistakes or form wrong opinions because they have not been trained to see many aspects of the object being studied or the situation in which they find themselves. To help develop a sense of alertness and a feeling that close attention must be given to demonstrations or other situations, the following techniques have been found valuable:

1. Near the beginning of the semester, arrange six or eight articles on a table or desk. Ask the students to study the display for one minute. Cover the display and then ask each member of the class to describe in as much detail as possible as many articles as he can. Throughout the semester repeat the test, increasing the number of items displayed.
2. At the beginning of a class period give the class the following oral instruction: "A person is said to be observant when he is conscious of what is happening about him or what goes on in situations which he meets. Today we want to study your ability to see what goes on in a situation. I have set up a simple apparatus and will perform an easy demonstration. When I have completed the demonstration you will be given an opportunity to write down the different things which I did and which took place during the demonstration."

Without further comment start on the demonstration using some apparatus such as is indicated below.



- (1) Boil a *little* water in the *round* flask.
- (2) Remove from the fire and quickly insert a glass tube with a narrow tip and a rubber hose.
- (3) Insert the hose into a *beaker* of water.
- (4) The reduced air pressure inside the flask causes the water in the beaker to be pushed up into the flask in the form of a *spray*.
- (5) When the spray stops, place the flask on the table, remove the stopper and pour out the liquid, and turn off the gas.

- (6) Distribute the following questions to the students or ask them orally:
- a. The liquid in the flask was boiled to dryness.
 - b. At the beginning of the experiment before the water began to boil:
 - _____ Tiny bubbles formed in the water.
 - _____ Drops of water collected on the outside of the flask.
 - _____ Steam came from the flask.
 - c. After the water began to boil, the teacher:
 - _____ Poured out the water.
 - _____ Inserted a stopper.
 - _____ Inserted a stopper with a rubber hose attached.
 - _____ Inserted a stopper and a glass tube with a hose on the end.
 - d. After the teacher inserted the hose into the beaker of water:
 - _____ The level of water rose in the beaker.
 - _____ The level of water fell in the beaker.
 - _____ More water came into the round flask.
 - e. At the close of the demonstration the teacher did the following things: (List the order in which these things were done by numbering them from 1-4.)
 - _____ The gas was turned off.
 - _____ The liquid was poured out.
 - _____ The stopper was removed.
 - _____ The flask was placed on the table.

After each of the techniques has been used, the teacher should note those problems which students failed because the questions were improperly constructed. Corrections should be made before the next administration of the test. As items become perfected, they may be filed for future use. Over a period of a few years there will accumulate sufficient items to permit a wide choice to fit the needs of classes of different caliber or different methods of instruction.

SUMMARY

A program for broadening the scope of science evaluation in secondary schools has been suggested and described. Four specific areas for evaluation, normally neglected in the schools, have been illustrated. These were: (1) speed and understanding in reading science materials, (2) resourcefulness in the use of original apparatus, (3) interpreting data from experiments and charts and graphs, and (4) ability to observe.

The need for pupils trained in mathematics, physics, chemistry, and biology—the languages of modern civilization—stands out in bold relief. The percentage of enrollments in these subjects is decreasing each year and the demand for such training is increasing.

Snow shovel on wheels is designed to take the strain out of removing snow from walks and driveways. The 30-inch blade on this manual snow plow can be adjusted easily to push snow to the right or left, or forward. The plow is of sturdy steel construction and rolls easily on rubber tires.

COURSE PATTERNS IN MATHEMATICS STUDIED BY HIGH SCHOOL STUDENTS¹

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Most of today's public secondary schools offer their students a wide choice when selecting the curriculum which they wish to pursue. This is as true in the field of high school mathematics as it is in other areas. The writer recently completed a study of the mathematics course patterns which had been studied by 1227 high school seniors (from forty Iowa public high schools), and seventy-eight different mathematics course patterns were identified when individual course names were retained. After grouping all courses under more general headings (e.g., algebra, four semesters, instead of elementary algebra, two semesters, advanced algebra, two semesters) twenty-two distinct course patterns, each of which had been followed by nine or more students, were found. To facilitate classification courses labeled "shop mathematics," "business mathematics" "business arithmetic," etc., were grouped with the course labeled "consumer mathematics," and the entire pattern was called "Vocational and Consumer Mathematics."

The research was concerned with course patterns in mathematics and the degree of functional competence² that students achieve which is characteristic of studying a particular course pattern.³ Since functional competence is influenced by student ability as well as by the type of instruction in mathematics, it was necessary to control the influence of ability in measuring functional competence. Therefore, it seemed that values for two variables were necessary. These variables are the student's ability and "level" of functional competence in mathematics.

The most widely accepted measure of student ability is the intelligence quotient (I.Q.). The measure of student ability used in this study, however, is the composite score on the *Iowa Tests of Educational Development*.⁴ This measure of ability was used because it was used because it was available for all the students who participated in the research, and a study done by the Examination Service of the State University of Iowa shows that there are significant correla-

¹ This article is based on research done by the author at the State University of Iowa under the direction of Professor H. Vernon Price and Professor L. A. Van Dyke.

² As defined by the "Second Report of the Commission on Post-War Plans," *The Mathematics Teacher*, 38: 195-221, May, 1945; and measured by the *Davis Test of Functional Competence in Mathematics*, published by the World Book Co., Yonkers-on-Hudson, New York, 1951.

³ The writer is aware that courses in related areas, e.g., physics and bookkeeping, could have an effect upon the degree of functional competence a student attains in mathematics.

⁴ Published by Science Research Associates, Inc., 57 W. Grand Ave., Chicago 10, Illinois, 1953.

tions⁵ between the composite score and measures of scholastic aptitude.

The arithmetic mean (in standard score units) of the scores which the students in each course pattern received on the Davis test was interpreted to mean the level of functional competence which had been achieved by those students. The arithmetic mean (in standard score units) of the composite scores on the Iowa tests received by the students in each course pattern was used as a measure of their scholastic aptitude. The factor of scholastic aptitude was controlled by the method of the analysis of covariance, and the Davis test means were adjusted accordingly. The separate course patterns which were found, the number of students who studied each course pattern, the mean score (in standard score units) on the Iowa tests for each pattern (\bar{I}), the mean score (in standard score units) on the Davis test for each pattern (\bar{D}), and the adjusted mean score (in standard score units) for the Davis test that resulted for each pattern after scholastic aptitude had been statistically controlled (Adj. \bar{D}) are presented in Table I. The number following each course represents the number of semesters the subject had been studied, e.g., "Algebra 2" means that the students in that pattern have completed two semesters of algebra.

The following are supported by the data contained in Table I.

1. In most cases the students whose adjusted mean scores on the Davis test were highest enrolled in those course patterns covering the greatest number of semesters.
2. In most cases the students enrolled in the course patterns covering the greatest number of semesters of mathematics study achieved the highest scores on the Iowa tests.
3. The most popular course pattern among the students who participated in the study is two semesters of algebra and two semesters of geometry (Pattern N).

Table II lists for each category (i.e., the number of semesters of mathematics studied) the number of students, the Iowa tests mean (\bar{I}), the Davis test mean (\bar{D}), the Davis test mean after scholastic aptitude had been statistically controlled (Adj. \bar{D}), and the results of the analysis of covariance. When a value of the F-ratio is listed in Table II as being significant, (i.e., $P \leq 5\%$), it should be interpreted to mean that the students in at least one course pattern in the category have achieved a level of measured functional competence that is significantly better than the level achieved by students in at least one other course pattern in the same category. The data listed in Table II allow the hypothesis of no difference in the course-pattern

⁵ State University of Iowa, University Examination Service, Technical Bulletin No. 4, *Relationships Among the Tests Given to Students Who Enter the College of Liberal Arts, Iowa*. City, Iowa, April, 1949, p. 2.

means to be accepted for the three-and-four-semester categories and rejected for the two-, five-, six-, and seven-semester categories. For the latter categories t-tests were made between the adjusted Davis test means for all possible combinations of course patterns within each category. The results of these t-tests, in terms of the significance of the difference between the adjusted Davis test means of any two course patterns, will be found in Table III. (The course patterns are referred to in Table III by the same letter they were assigned in Table I.) It will be noted that sixteen significant differences were found.

TABLE I
MAJOR COURSE PATTERNS FOR STUDENTS IN THE STUDY

Course Patterns	No. of Semesters	No. of Students	\bar{I}	\bar{D}	Adj. \bar{D}
A Algebra 4; Geometry 3	7	89	25.06	145.48	144.07
B General Math. 2; Algebra 3; Geometry 2	7	20	19.20	133.05	139.88
C Algebra 4; Geometry 2; Trigonometry 1	7	10	24.90	143.50	142.32
D Algebra 3; Geometry 2; Trigonometry 1	6	40	23.60	143.05	140.80
E General Math. 2; Algebra 2; Geometry 2	6	18	20.83	130.44	131.12
F Algebra 3; Geometry 3	6	15	22.40	138.07	136.69
G Algebra 3; Geometry 2; Vocational and Consumer Math. 1	6	12	19.08	132.25	135.24
H Algebra 4; Geometry 2	6	10	22.60	135.10	134.46
I General Math. 2; Algebra 2; Vocational and Consumer Math. 2	6	9	12.22	111.11	123.12
J Algebra 2; Geometry 2; Vocational and Consumer Math. 1	5	94	19.26	124.35	125.26
K Algebra 3; Geometry 2	5	67	22.46	136.12	133.03
L General Math. 2; Algebra 2; Vocational and Consumer Math. 1	5	23	15.22	109.61	115.58
M General Math. 2; Algebra 2; Geometry 1	5	12	21.17	136.17	134.69
N Algebra 2; Geometry 2	4	284	19.30	121.32	119.13
O General Math. 2; Algebra 2	4	92	16.45	119.10	121.70
P General Math. 2; Vocational and Consumer Math. 2	4	21	13.52	114.57	121.30
Q General Math. 2; Algebra 1; Vocational and Consumer Math. 1	4	10	17.10	114.50	116.19
R General Math. 2; Vocational and Consumer Math. 1	3	78	13.50	113.40	115.09
S Algebra 2; Vocational and Consumer Math. 1	3	73	15.85	115.29	114.37
T Algebra 2; Geometry 1	3	13	19.46	122.62	117.68
U Algebra 2	2	142	16.92	116.56	114.56
V General Math. 2	2	95	13.23	107.94	110.47

Any course pattern whose adjusted Davis test mean was significantly better than the adjusted Davis test mean of at least one other course pattern within the same semester category will be found in Table IV. Table IV shows that eleven of the sixteen significant differences (which were listed in Table III) are in favor of the conventional⁶ mathematics course patterns. These conventional course patterns, i.e., A, D, F, H, K, and U (see Table IV), also enroll over one-half (363 of 656 students) of the students in the seven-, six-, five-, and two-semester categories. Five of the sixteen significant differences which were found between the adjusted Davis test means for all combinations of course patterns within the same semester category (see Table III) can be credited to the non-conventional⁷ mathematics course patterns. These course patterns are E, G, J, and M (see Table III). These course patterns enrolled 136 of the total 656 students (approximately 20 per cent) in these four-semester categories. Consequently, the conventional mathematics course patterns in these semester categories (i.e., the seven-, six-, five-, and two-semester categories) seem to be more effective than most of the nonconventional mathematics course patterns with respect to providing those students who study them with the knowledge of mathematics which

TABLE II
RESULTS OF THE ANALYSIS OF COVARIANCE BETWEEN COURSE PATTERNS
WITHIN A GIVEN NUMBER OF SEMESTERS

No. of Semesters	No. of Students	\bar{I}	\bar{D}	Adj. \bar{D}	Value of the F-ratio	Degrees of Freedom	Level of Significance (P)
Seven	119	24.06	143.23	135.07	3.079	2;115	$P = 5\%$
Six	104	21.35	135.38	131.13	6.685	5; 97	$P < 5\%$
Five	196	19.99	127.37	125.08	21.058	3;191	$P < 5\%$
Four	407	18.30	120.30	120.44	1.398	3;402	$P > 5\%$
Three	164	15.02	114.97	119.84	0.570	2;160	$P > 5\%$
Two	237	15.44	113.10	117.37	12.583	1;234	$P < 5\%$

is necessary to increase their level of achievement of functional competence (as measured by the Davis test). Perhaps the superiority of the achievement of those students enrolled in the conventional mathematics course patterns could be due to the fact that in most cases students with the greatest scholastic aptitude enroll in these course patterns (the mean composite score on the Iowa tests, I, listed in Table I for each course pattern supports this statement), and it is possible that these students have better memories.

⁶ The reader should interpret the phrase "conventional mathematics course patterns" to mean those course patterns where the students study algebra in the ninth grade, geometry in the tenth grade, and advanced algebra, solid geometry, and trigonometry in the eleventh and twelfth grades.

⁷ Those course patterns which contained general mathematics and/or vocational and consumer mathematics.

The data given in Table I and Table III indicate that when students study general mathematics as a basic course in the ninth grade and pursue the study of the conventional courses (i.e., algebra, geometry, and trigonometry), these students achieve as high a degree of measured functional competence as do those students who study algebra in the ninth grade and follow some of the conventional mathematics course patterns. This statement is supported by the fact that no statistically significant differences were found between the adjusted Davis test means for the following course patterns: B and C, E and G, E and H, K and M, and N and O. Consequently, it is believed that the level of functional competence (as measured by the

TABLE III
RESULTS OF TESTING THE NULL HYPOTHESIS BETWEEN COURSE PATTERNS
IN MATHEMATICS

Category	Course Patterns Between Which Significance is Being Tested	Standard Deviation	t-value	Level of Significance (P)	Pattern Which Significant Difference Favors
Seven Semesters	A & B	1.729	2.423	1% < P < 2%	A
	A & C	2.156	0.810	40% < P < 50%	—
	B & C	2.581	0.945	30% < P < 40%	—
Six Semesters	D & E	2.269	4.267	P < 1%	D
	D & F	2.383	1.725	5% < P < 10%	—
	D & G	2.665	2.087	2% < P < 5%	D
	D & H	2.775	2.285	2% < P < 5%	D
	D & I	3.362	5.260	P < 1%	D
	E & F	2.759	2.019	2% < P < 5%	F
	E & G	2.933	1.405	10% < P < 20%	—
	E & H	3.106	1.075	20% < P < 30%	—
	E & I	3.355	2.315	2% < P < 5%	E
	F & G	3.076	0.471	60% < P < 70%	—
	F & H	3.202	0.697	40% < P < 50%	—
	F & I	3.646	3.722	P < 1%	F
	G & H	3.394	0.230	80% < P < 90%	—
	G & I	3.603	3.364	P < 1%	G
	H & I	3.926	2.889	P < 1%	H
Five Semesters	J & K	1.470	5.289	P < 1%	K
	J & L	2.200	4.400	P < 1%	J
	J & M	2.820	3.344	P < 1%	M
	K & L	2.468	7.071	P < 1%	K
	K & M	2.876	0.577	50% < P < 60%	—
	L & M	3.358	5.691	P < 1%	M
Two Semesters	U & V	1.241	3.538	P < 1%	U

Davis test) which students achieve is more dependent upon the courses studied beyond the ninth grade than it is upon the course studied in the ninth grade.

The F-test yielded a value that was not statistically significant at the five per cent level of confidence for the three- and four-semester categories. Therefore, it can be said that all the mathematics course patterns within these categories are equally effective with regard to providing those students who study them with the elements of functional competence necessary to perform equally well on the Davis test. It can be seen, by examining Tables II and III, that in the two-semester category the students who study only algebra score higher on the Davis test than do the students who study only general mathematics.

In the five-, six-, and seven-semester categories there seemed to be a tendency for those students who had studied geometry, and in some cases trigonometry, to achieve higher levels of functional competence (as measured by the Davis test) than those students who

TABLE IV

COURSE PATTERNS WHOSE ADJUSTED DAVIS TEST MEANS ARE SIGNIFICANTLY BETTER THAN THE ADJUSTED DAVIS TESTS MEANS OF AT LEAST ONE OTHER COURSE PATTERN WITHIN THE SAME SEMESTER CATEGORY

Semester Category	Course Pattern	No. of Times Course Pattern Adjusted Davis Test Mean Was Superior
Seven Semesters	A: Algebra 4; Geometry 3	1
Six Semesters:	D: Algebra 3; Geometry 2; Trigonometry 1	4
	G: General Math. 2; Algebra 2; Geometry 2	1
	F: Algebra 3; Geometry 3	2
	G: Algebra 3; Geometry 2; Vocational & Consumer Math. 1	1
	H: Algebra 4; Geometry 2	1
Five Semesters:	J: Algebra 2; Geometry 2; Vocational & Consumer Math. 1	1
	K: Algebra 3; Geometry 2	2
	M: General Math. 2; Algebra 2; Geometry 1	2
Two Semesters:	U: Algebra 2	1

had not studied geometry. This "need" for a study of geometry⁸ seemed to increase as the number of semesters of mathematics which were studied increased, but the evidence was not sufficient to justify drawing any conclusions. Perhaps this "tendency" was spurious or a function of the measuring instrument used, i.e., the Davis test. However, if this point were investigated, mathematics teachers would better understand the contribution which geometry makes to the level of functional competence which students achieve.

From the results of this study the writer believes that four definite conclusions can be drawn.

1. For the Iowa high school students who study only two semesters of mathematics the study of algebra produces results which are significantly better than those produced by general mathematics.
2. General mathematics is as effective as is algebra as a ninth grade course if the study of mathematics is continued in the higher grades.
3. Students in Iowa public high schools who study the more "conventional" mathematics course patterns score higher on the Davis test than students who study the "unconventional" mathematics course patterns.
4. A student's level of achievement of functional competence depends more upon the mathematics courses which are studied in grades ten, eleven, and twelve than it depends upon the course studied in the ninth grade.

⁸ The writer is aware, since the results were not entirely consistent, that this apparent "need" could be a function of the measuring instrument which was used, i.e., the Davis test.

FORMULAS FOR BETTER READING IN MATHEMATICS

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When mathematics teachers say "If he could read, I could teach him mathematics" they are implying that general reading ability is the panacea of all subject matter woes. Nothing could be more foreign to the truth. We know, for example, that many good readers read rapidly, employing, in some instances, a kind of telegraphic technique of omitting non-essential words. This is an excellent method for reading easy prose but has no place in reading mathematics. Since in reading mathematics, every word is as important as a marriage contract, the teacher must stress the need for slow, metic-

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ulous reading. The following approaches have been used successfully by teachers who wanted to combat these skimming tendencies.

1. One teacher gave students training in spotting irrelevant sentences and phrases which had been inserted in problems drawn from supplementary texts.

2. Another teacher used non-mathematical materials for training purposes. His students were given exercises in deciphering coded messages which fellow classmates had worked out. Training also was given in reading newspaper want-ad sections for specific facts and information.

3. Many teachers believe that the most valuable training in careful reading grows out of having students approximate an answer before trying to work out a detailed solution.

Much of the difficulty in reading mathematical material can be attributed to its vocabulary burden. Technical words like "quotient, integer, exponent, and addend" must be clearly understood if a problem employing one or more of them is encountered. In addition to a technical vocabulary, many common words having a special mathematical connotation figure in mathematical problems. Words like "revolution, base, and plane," for example, are but a few which must be understood if a problem employing them is to prove meaningful. There are many ways to help students master their mathematical vocabulary. The following have proved most helpful.

1. Organize word rummy games using cards which list the technical vocabulary you wish your students to master. One of the best listings of technical terms indispensable to reading meaningfully in mathematics can be found in Cole's *The Teacher's Handbook of Technical Vocabulary*.¹

2. Have vocabulary down games similar to the old fashioned spell down game so popular in elementary school classrooms.

3. With the help of your class, work out a glossary of terms which can be used by them and then handed down to subsequent classes.

4. Make use of visual aids of all kinds. This is valuable in building concepts which make terms meaningful.

5. Write a base word on the board and then ask students to build a list of terms falling in that category. Provide your students with clues by listing the first and last letter of the words along with spaces representing intervening letters. The following is an example.

TRIANGLES
R — — — T
I — — — — — S
E — — — — — L

¹ Cole, Luella, *The Teacher's Handbook of Technical Vocabulary*, Public School Publishing Company, Bloomington, Illinois, 1940.

6. For teaching important formulas, play a form of bingo. Call the game FORMULO. Select twenty-five important formulas and substitute them for the numbers used in the ordinary game of bingo. Six cards of twenty-five formulas constitute a set. All cards include the same formulas but these appear in a different order on each. Descriptions of the formulas are then written on slips of paper. Each player is given a card. Slips are then drawn at random and read. If a slip reads, for example, "area of a rectangle," the players then would look for the formula " $L \times W$." Each formula called is covered with a small paper square until one row, either vertical, diagonal, or horizontal, has been completed. The individual holding this card is the winner.

Other suggestions of a miscellaneous nature which have proved helpful in reading mathematics more efficiently are these:

1. When a student is confused and can't solve a problem, have him read it out loud to determine if he is reading it correctly.

2. Try reading a problem orally for your class and ask your students to write a version of how they would go about solving it. Then have the students share their versions of the solution.

3. When students are bewildered by mathematical concepts, have them read comparable lessons in an easier, supplementary text.

4. When problems are long and involved encourage students to re-write them employing short, simple sentences.

5. Devise training exercises designed to develop the reading skills students need to read mathematics meaningfully. For example, a series of exercises based on "What information does the problem give you?" or "What are you asked to find?" would prove very helpful in teaching students to read mathematical problems more intelligently.

6. With discrimination, choose some of your better readers to work with the poorer ones individually, or in groups. Experiments have shown that an arrangement such as this pays dividends. The teacher is relieved of time consuming responsibility and the students learn as much if not more than they would working directly under his supervision.

The mathematics teacher can do a great deal to help students read mathematics more efficiently. He must realize, however, that a few students are so severely retarded in their basic reading skills that they require remedial as well as the developmental help he can give them.

"We shall never use our strength to try to impress upon another people our own cherished political and economic institutions."

—DWIGHT D. EISENHOWER

THE CRYSTAL SET—A CENTER OF INTEREST

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Areas having to do with electricity and communications arouse great interest among boys of the intermediate and upper grades. In my own classroom I have observed that these types of activities are unique as a motivating force for both the slow learners and the gifted youngsters.

The building of a crystal radio set is among the most popular of such activities. The crystal set lends itself to classroom use because it is inexpensive and simple to make.

Following is a description of the parts and materials needed:

1. The antenna. This may be about 25 feet of insulated or un-insulated copper wire, connected to the set, stretched at a distance above it. It may be strung outdoors through a window to a pole or tree, or inside along the classroom wall. The antenna picks up the radio waves sent from the transmitting station. If un-insulated wire is used, there should be a lightning arrester connected between the set and the antenna wire.
2. The ground wire. This may be insulated or un-insulated copper wire. It may be connected from the set to any metal object which directly or indirectly leads to the earth. The radiator in the classroom makes a good ground connection; a pipe placed in the ground outside the classroom is also satisfactory.
3. Coil. The coil consists of insulated copper wire wound around a cylinder form. No. 22 or No. 24 magnet wire is used. An average-sized oatmeal box works well.
4. Tuning condenser. This device is made of metal leaves or plates, some of which are stationary, and some of which turn when one rotates the knob which protrudes from them. The part that is stationary is called the stator; the part that turns is called the rotor. It is the tuning device in radios, and it acts in conjunction with the coil in this respect. A small one having 17 to 21 plates is desirable.
5. Fixed condenser. This type of condenser is called "fixed" because it is not moved or turned like the tuning condenser. Fixed condensers are usually small and may be rectangular or tubular in shape. For this set one marked .00025 micro-fards (mfd.) is needed.
6. Germanium crystal. This is a small piece of germanium on which the manufacturer has fixed or located the spot most sensitive to vibrations. Hence, it is sometimes referred to as a fixed crystal. The one best suited to this set is designated as the 1N34 type.

7. Insulated copper wire for coil and connections. As already mentioned, No. 22 or 24 magnet wire is used. This can be obtained in $\frac{1}{2}$ pound spools, which contain more than enough wire for the coil and connections.
8. Clips and screws. For the connections are needed three brass clips known as Fahnestock clips, and five small wood screws, size No. 6.
9. Mounting board. A piece of plywood about a foot square will hold all the parts.
10. Earphones.
11. Soldering iron or woodburning set.
12. Solder. This is a wire-like combination of tin and lead with a low melting point, for securing connections.

These parts need not be new. If a teacher inquires about such parts among the pupils in his class, he will find invariably a few individuals who have such gear. Some of the parts, such as the tuning condenser and fixed condenser, may be obtained from old, discarded radios. Some boys enjoy dismantling old radios and salvaging parts.

The local radio and television repairman is a good source of used radio parts, wire, etc. Usually he is cooperative in letting the pupils have old gear that he plans to discard. He is a valuable source of information on the subject of radio for both the teacher and pupils.

There are several radio wholesale houses in the country from whom one can obtain catalogues free of charge. Parts are listed at lower prices in these catalogues. Some slow learners who dislike writing business letters seem eager to write to these companies for these free catalogues.

PROCEDURE

It is best to plan to use the set near a window since this is most convenient for the antenna and ground connections.

Stage One:

Remove lid and bottom from oatmeal box so that it is open at both ends. Apply several coats of shellac or enamel paint to box in order to stiffen it.

With a nail punch two holes about 1 inch from the edge of box. Place holes about $\frac{1}{4}$ inch apart. Do the same thing at other end of box.

Put the No. 22 or 24 wire from the spool through one hole, then the other hole, leaving a piece of about $1\frac{1}{2}$ feet dangling. Wind wire very close together around box. Turns must not be on top of one another; however, there must not be any space between them. Wind wire until you have reached the other end of the box. Put wire through holes at this end. Cut off wire from spool, leaving about one foot dangling.

There should be about 130 turns of wire on box. Shellac again so that wire will stay in place. Wait till shellac has dried.

Mount coil horizontally at one end of the board, using a wood screw on each side. (No. 1 in pictorial drawing.) Place board so that coil is toward the back as you face it. Facing the board now, the right end of the coil will be for the antenna connection; the left end will be for the ground.

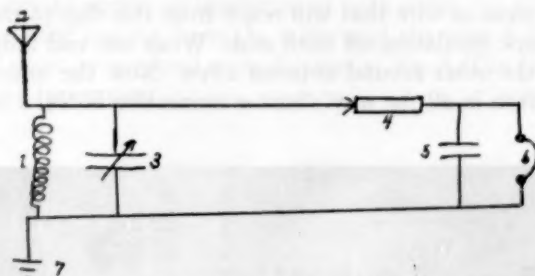


FIG. 1. Schematic diagram. 1. Coil. 2. Antenna. 3. Tuning condenser. 4. Crystal. 5. Fixed condenser. 6. Ear phones. 7. Ground.

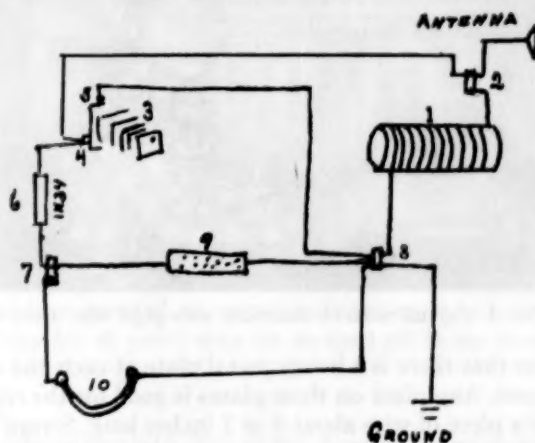


FIG. 2. Pictorial diagram. 1. Coil. 2. Antenna connection. 3. Tuning condenser. 4. Stator connection. 5. Rotor connection. 6. Crystal (1N34). 7 & 8. Fahnestock clips. 9. Fixed condenser. 10. Ear phones.

Stage Two:

Antenna connection: Place one of the Fahnestock clips to right of coil, near edge of board. Screw it in only part way. (No. 2 in pictorial drawing.)

Bring wire from right side of coil to this screw. Cut off excess wire. Scrape off insulation, and wrap it around screw.

Stage Three:

Tuning condenser: Mount the tuning condenser with two wood screws, about four inches from left side of coil. (No. 3 in pictorial drawing.)

Note that on the back of the tuning condenser is a metal flap with a hole in it. This is the *stator* lead, and has to be connected to the antenna screw.

Take a piece of wire that will reach from this flap to the antenna screw. Scrape insulation off both ends. Wrap one end around metal flap, and the other around antenna screw. Now the antenna screw may be driven in all the way. (Stator connection is No. 4 in pictorial drawing.)



FIG. 3. Crystal radio in classroom with pupil who made it.

Note also that there is a heavy metal plate at each end of the tuning condenser. Any place on these plates is good for the *rotor* connection. Take a piece of wire about 6 or 7 inches long. Scrape insulation off one end; solder it anywhere on one of the heavy metal pieces. Leave the other end of this wire free for the moment. (Rotor is No. 5 on pictorial drawing.)

Note: Tuning condensers out of old radios may have more than one section of plates. Hook up only one of these sections.

Stage Four:

Crystal: This has two wires coming out of it. Wrap one end around the stator flap. Leave other wire free for the moment. (No. 6 on pictorial drawing.)

Stage Five:

Clips: At the front of broad, directly opposite coil, mount the other two Fahnestock clips, about 3 or 4 inches apart. Screw in only part way, (No. 7 and 8 on pictorial drawing) There are three free wires which must be connected to these screws. First connect the free end of the crystal to the screw at No. 7. If this wire from the crystal is too short, attach to it another piece of wire long enough to reach the screw. Be sure to scrape off insulation at both ends.

Both the wire coming from the rotor and the wire from the left side of coil must be wrapped around screw at No. 8. The connections at No. 8 are for ground.

Stage Six:

Fixed condenser: This has two wires coming from it. Wrap one wire around screw at No. 7 and the other at No. 8. Now force screws at 7 and 8 all the way in. (No. 9 on pictorial drawing.)

Stage Seven:

Solder all connections that have been made. When soldering crystal connections, hold crystal gently with a pair of pliers. Heat harms the germanium of which the crystal is made. The pliers will absorb the heat from the soldering iron.

Stage Eight:

Set up the wire for the antenna. Hook one end of the wire to Fahnestock clip at right of coil. If insulated wire is used for antenna, be certain that the part hooked into clip has been scraped off.

Set up wire for the ground. Lead the wire from the radiator or pipe to the clip at No. 8. Be certain insulation is removed from part which is hooked into clip, and also from part touching radiator. Also, if radiator has been painted, an unpainted spot must be found for contact, or a tiny bit of paint may be scraped off in an inconspicuous place.

Stage Nine:

Place earphone tips into clips at No. 7 and 8. Keep moving tuning condenser until a station is heard.

I have included a diagram of schematic symbols together with the pictorial drawing. Pupils seem to be fascinated by these symbols, and they take pride in learning them.

There are, of course, other ways to build a crystal radio receiver. Once pupils get started on this activity, they will come up with variations. Some become interested enough to make radio a hobby.

Truly, I have found this experience extremely worthwhile in making the study of modern communications more meaningful to pupils.

CONFUSION RESULTING FROM DUPLICATION OF SYMBOLISM AND DEFINITIONS IN MATHEMATICS

E. A. HABEL

Pensacola Junior College, Pensacola, Florida

The opinion is widely held that the deficiencies of college freshmen with mathematics are primarily a result of lack of understanding of requisite mathematical concepts. Research by the author seems to indicate¹ that in many instances failure to understand simple mathematical notation is also the source of much error and that the fault lies not alone with teacher and student but in part with the structure of mathematics and the textbooks which expound it.

Mathematics has been defined by some people as the science of accuracy and precision, yet mathematics, as far as many of its commonly accepted definitions and symbols are concerned, is not itself exactly scientific. In any exact science, in order to avoid confusion, each entity has one name and one symbol, just as with the elements in chemistry. Consequently, when a chemist deals with hydrogen or oxygen, it is always "H" or "O." But not so if a mathematician is dealing with a fraction! He may call it by the various names of ratio, rate, or indicated division. Similarly, division may and does go under the pseudonym of factoring. Or, if it is required of the young student to find a "derivative," then, depending upon the authority, he may seek $D_x y$, y' , dy/dx , $f'(x)$, etc.

Such multiplicity of notation and definition has no benefit whatsoever for the beginner, contributes to his confusion and his mistakes and succeeds in making a difficult subject incomprehensible to him. The student who is left to find out for himself that a dot, a cross mark, parentheses jammed against one another, or two letters adjacent to one another all are equivalent symbols meaning multiply may become bewildered in the early stages of study—especially when the dot for multiply so closely resembles the dot for decimal point or the cross mark the "x" for the unknown or variable in algebra.

Findings from an attempt to diagnose the causes of repetitive errors² of college freshmen have led the author to believe that the needless multiplicity of definition and symbolism increases the mathematical ills of students. This short article is a plea for clarification of our symbolism so that there need be no doubt in anyone's mind as to the order and kind of operation intended in such important little problems as $\frac{1}{2} \cdot \frac{3}{4} - \frac{1}{2} \cdot \frac{3}{4}$.

¹ E. A. Habel, *An Experiment in the Diagnosis and Remedy of Errors of College Freshmen in Arithmetic and Radicals*, *SCHOOL SCIENCE AND MATHEMATICS*, 51: 105-113, February, 1951.

² *Ibid.*

PROBLEM DEPARTMENT

CONDUCTED BY MARGARET F. WILLERDING

Harris Teachers College, St. Louis, Missouri

This department aims to provide problems of varying degrees of difficulty which will interest anyone engaged in the study of mathematics.

All readers are invited to propose problems and to solve problems here proposed. Drawings to illustrate the problems should be well done in India ink. Problems and solutions will be credited to their authors. Each solution or proposed problem sent the Editor should have the author's name introducing the problem or solution as on the following pages.

The editor of the Department desires to serve his readers by making it interesting and helpful to them. Address suggestions and problems to Margaret F. Willerding, Harris Teachers College, St. Louis, Missouri.

SOLUTIONS AND PROBLEMS

Note. Persons sending in solutions and submitting problems for solutions should observe the following instructions.

1. Solutions should be in typed form, double spaced.
2. Drawings in India ink should be on a separate page from the solution.
3. Give the solution to the problem which you propose if you have one and also the source and any known references to it.
4. In general when several solutions are correct, the one submitted in the best form will be used.

2641. Proposed by Cecil B. Read, University of Wichita.

Find, without using logarithms, the limit as x approaches infinity of $2^x/e^{x^2}$.

Solution by Charles H. Butler, Kalamazoo, Michigan

Since $e > 2$ there exists a number a ($a > 1$) such that $2a = e$. Set $e^x = 2^x \cdot a^x$.

Note that

$$\begin{aligned} e^{x^2} &= e^{a^x \cdot x} = e^a \cdot e^{x^2-x} = 2^x \cdot a^x (e^{x^2-x}) \\ \therefore \frac{2^x}{e^{x^2}} &= \frac{2^x}{2^x a^x (e^{x^2-x})} = \frac{1}{a^x (e^{x^2-x})} \end{aligned}$$

Now since $a > 1$ and $e > 1$, it follows that as x becomes infinite, (1) a^x becomes infinite; (2) $x^2 - x$ becomes infinite; and (3) e^{x^2-x} becomes infinite. Thus

$$\lim_{x \rightarrow \infty} a^x (e^{x^2-x}) = \infty.$$

Therefore

$$\lim_{x \rightarrow \infty} \frac{2^x}{e^{x^2}} = \lim_{x \rightarrow \infty} \frac{1}{a^x (e^{x^2-x})} = \frac{1}{\infty} = 0.$$

Solutions were also offered by C. W. Trigg, Los Angeles, Calif.; Alan Wayne, Baldwin, N. Y., and the proposer.

2462. Proposed by Julian H. Braun, U. S. Army, Aberdeen Proving Ground, Md.

In a certain college F per cent of the freshman class are men having 1-A draft classification. A directive exempts from the draft those "1-A's" who are in the upper U per cent of their class. The army calls all 1-A men available under the above restriction. Now, of course, after the first call, some 1-A students who were in the upper U per cent are then beneath the dividing line in the reduced

class. Supposing the army continues to call men as far as it can go under this system, in what upper percentile group of the original class would a 1-A student have to be in order that he be exempted from the draft? (Assume a uniform percentile distribution of 1-A's on the ranking scale of the original class.)

Solution by the proposer

An obvious method of solution would be to employ infinite series. However this would be tedious. A simpler method of solution is based on a consideration of the situation that must exist when the Army has called all the men it can. Thus:

Solution to the "draft problem" by the proposer

Let x = required percentile group. When the Army has called all the men it can, then the x group will consist of $F\%$ 1-A's and $(100-F)\%$ non 1-A's. Thus the percentage of the class who are 1-A's not called is $(F/100)x$, and it follows that the size of the reduced class is then $[100-F+(F/100)x]\%$ of the original size. But the upper $U\%$ of the reduced class must equal $x\%$ when the Army has called all the men it can. Thus:

$$x = (U/100)[100 - F + (F/100)x].$$

Solving

$$x = \frac{(100-F)U}{100-FU/100}.$$

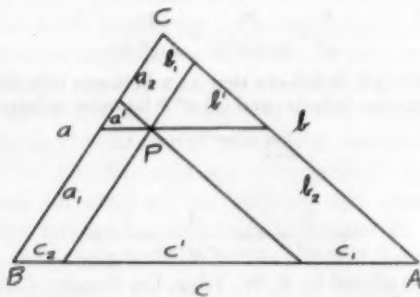
2463. Proposed by Richard H. Bates, Milford, N. Y.

Through any interior point P in triangle ABC , lines are drawn parallel to the three sides of the triangle, dividing these sides into three segments each. If the center segments of sides a , b , and c are denoted by a' , b' , and c' respectively, show that

$$a'/a + b'/b + c'/c = 1.$$

Solution by C. W. Trigg, Los Angeles City College

From the parallelograms and similar triangles in the figure, we have $a'/a = m/b = b_1/b$ and $a'/a = n/c = c_2/c$.



In like manner,

$$b'/b = c_1/c, \quad b'/b = a_2/a, \quad c'/c = b_2/b, \quad c'/c = a_1/a.$$

These, together with the identities $a'/a = a'/a$, $b'/b = b'/b$, and $c'/c = c'/c$ constitute nine equations which when added together give

$$3(a'/a + b'/b + c'/c) = (a_1 + a' + a_2)/a + (b_1 + b' + b_2)/b + (c_1 + c' + c_2)/c = 3.$$

Therefore

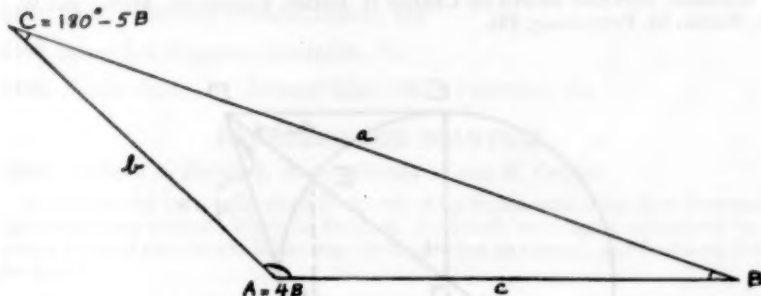
$$a'/a+b'/b+c'/c=1.$$

Solutions were also offered by Leon Bankoff, Los Angeles, Calif.; Charles H. Butler, Kalamazoo, Mich.; A. R. Haynes, Tacoma, Wash.; J. Byers King, Denton, Md.; Alan Wayne, Baldwin, N. Y.; and the proposer.

2464. Proposed by C. W. Trigg, Los Angeles City College

In triangle ABC , if $\angle A = 4\angle B$, find the relationship involving the sides. Show that, in particular, if $b=c$, then $a=b\sqrt{3}$.

Solution by Charles H. Butler, Kalamazoo, Michigan



From the law of sines

$$\frac{b}{\sin B} = \frac{a}{\sin 4B} = \frac{c}{\sin (180^\circ - 5B)} = \frac{c}{\sin 5B},$$

whence

$$a = \frac{b \sin 4B}{\sin B} = \frac{c \sin 4B}{\sin 5B}.$$

In particular, if $b=c$, then $B=C=180^\circ-5B$, whence $B=30^\circ$. Therefore

$$a = b \frac{\sin 120^\circ}{\sin 30^\circ} = b \frac{\sqrt{3}}{2} \cdot \frac{2}{1} = b\sqrt{3}.$$

EDITOR'S NOTE: Alan Wayne, Baldwin, N. Y. points out that a generalization of this problem appeared in the *American Mathematical Monthly* January 1945, pp. 44-46, as Problem E 620.

Solutions were also offered by Richard Bracken, Pittsburgh, Pa.; Hattie Estey, Flint, N. Y.; A. R. Haynes, Tacoma, Wash.; J. W. Lindsey, Amarillo, Texas; W. R. Warne, St. Petersburg, Fla.; and Alan Wayne, Baldwin, N. Y.

2465. Proposed by C. W. Trigg, Los Angeles City College

In circle (O) central angle COE is less than 90° ; OE extended meets the circle again in A and the tangent at C and D . The parallel to CD through A meets the circle in B . Express angle ABD in terms of angle COE .

Solution by the proposer

Draw EB . Then ABE is a right angle. Denote angle COE by y and angle EBD by x . Then angle $CDO = 90^\circ - y = \text{angle } DAB$, and angle $ADB = 180^\circ - (90^\circ - y) - (90^\circ + x) = y - x$. Now in triangle ABD , by the law of sines, we

STUDENT HONOR ROLL

The Editor will be very happy to make special mention of classes, clubs, or individual students who offer solutions to problems submitted in this department. Teachers are urged to report to the Editor such solutions.

Editor's Note: For a time each student contributor will receive a copy of the magazine in which his name appears.

For this issue the Honor Roll appears below.

2461, 2464, 2465, 2466. *Lee Dresden Goldberg, Hillside, N. J.*

2463. *Thelma Hackest, Denton, Md.*

2463. *Barbara King, Denton, Md.*

2463, 2466. *J. Richard Thomas, Denton, Md.*

2466. *Mary Lou Haggerty, Springdale, Pa.*

2464. *Noreen Bayly, Mt. Lebanon High School, Pittsburgh, Pa.*

PROBLEMS FOR SOLUTION

2485. *Proposed by Dwight L. Foster, Florida A. and M. College.*

A man started for a walk when the hands of his watch were coincident between three and four o'clock. When he finished, the hands were again coincident between five and six o'clock. What was the time when he started, and how long did he walk?

2486. *Proposed by Christos B. Clavas, New York, N. Y.*

Take the parabola $y^2 = x$. Join the origin, O , with any point, P , on the parabola. From P draw a perpendicular on which OP intersects the x -axis at the point G . Prove the $OG = 1/\sin^2 \theta$ where θ is the angle XOP .

2487. *Proposed by A. R. Haynes, Tacoma, Wash.*

Solve

$$\begin{cases} (x+a)^4 + (x+b)^4 = 17(a-b)^4 \\ (x+a) - (x+b) = (a-b) \end{cases}$$

for x .

2488. *Proposed by C. W. Trigg, Los Angeles City College.*

On one side of an angle $\theta < 90^\circ$ there is a point, P , at a distance a from the vertex of the angle. From P a perpendicular is dropped to the other side of the angle; from the foot of this perpendicular another perpendicular is dropped to the first side. This process is continued indefinitely. Show that the length of the broken line is $2a \csc \theta \cos^2 \theta/2$. Also show that when $\theta = \pi/4$ the length of the broken line is equal to the perimeter of the triangle formed by the first perpendicular and the sides of the angle.

2489. *Proposed by Julius S. Miller, New Orleans, La.*

A sphere, a hoop, a disc, and a cube, have equal masses. The sphere, hoop, and disc have equal radii. All four start from rest and roll (slide in the case of the cube) down a smooth incline. Which one wins the race?

2490. *Proposed by Paul D. Thomas, Norman, Okla.*

The distance from the origin to the tangent at a variable point of every curve of a family is directly proportional to the derivative of the arc length with respect to the abscissa of the variable point. Find the equation to the family and its envelope.

BOOKS AND PAMPHLETS RECEIVED

MORE MODERN WONDERS AND HOW THEY WORK, by Captain Burr W. Leyson. Cloth. 215 pages. 13.5×20.5 cm. 1955. E. P. Dutton and Company, Inc., 300 Fourth Avenue, New York 10, N. Y. Price \$3.50.

ELECTRO-MAGNETIC MACHINES, by R. Langlois-Berthelot, M.I.E.E., M.A.I.-E.E., *Chief Research Engineer for Production and Transformation Equipment at l'Electricité de France, and Professor of Electrical Engineering at Ecole Supérieure d'Electricité, Paris*. Translated and Revised in Collaboration with Lieut. Colonel H. M. Clarke, T.D., M.Sc., M.I.E.E., *Senior Lecturer in Electrical Engineering, University of London, King's College*. Cloth. 535 pages. 13×21.5 cm. 1955. Philosophical Library, Inc., 15 E. 40th Street, New York 16, N. Y. Price \$15.00.

MAGNETIC MATERIALS IN THE ELECTRICAL INDUSTRY, by P. R. Bardell, B.Sc., M.I.E.E., F. Inst. P. Cloth. 288 pages. 13.5×21 cm. 1955. Philosophical Library, Inc., 15 East 40th Street, New York 16, N. Y. Price \$10.00.

SOUND BARRIER, THE STORY OF HIGH-SPEED FLIGHT, by Neville Duke and Edward Lanchbery. Cloth. Pages xi+129. 12×18.5 cm. 1955. Philosophical Library, Inc., 15 East 40th Street, New York 16, N. Y. Price \$4.75.

THE MOBILE MANUAL FOR RADIO AMATEURS, by the American Radio Relay League. Paper. 352 pages. 15×24 cm. 1955. The American Radio Relay League, 38 La Salle Road, West Hartford, Conn. Price \$2.50.

FUNDAMENTAL CONCEPTS OF MATHEMATICS, Second Edition, by R. H. Moorman, *Professor and Chairman of the Department of Mathematics of Tennessee Polytechnic Institute, Cookeville, Tennessee*. Paper. Pages iii+92. 20.5×27 cm. 1955. Burgess Publishing Company, Minneapolis 15, Minn. Price \$2.75.

ARITHMETIC IN GENERAL EDUCATION, by Dewey C. Duncan, Ph.D., *Chairman, Department of Mathematics, East Los Angeles Junior College*. Paper. Pages vi+194. 13.5×21.5 cm. 1955. Wm. C. Brown Company, 915 Main Street, Dubuque, Iowa. Price \$2.25.

MONOGRAPHS ON TOPICS OF MODERN MATHEMATICS, Edited by J. W. A. Young. Paper. Pages xvi+416. 12.5×20.5 cm. 1955. Dover Publications, Inc., 1780 Broadway, New York 19, N. Y. Price \$1.90, Cloth \$3.95.

THEORY OF GROUPS OF FINITE ORDER, Second Edition, by W. Burnside, M.A., F.R.S., D.Sc. (Dublin), LL.D. (Edinburgh). *Professor of Mathematics at the Royal Naval College, Greenwich*. Paper. Pages xxiv+512. 12.5×20 cm. 1955. Dover Publications, Inc., 1780 Broadway, New York 19, N. Y. Price \$2.00, Cloth \$3.95.

HYDRODYNAMICS. A STUDY IN LOGIC, FACT, AND SIMILITUDE, by Garrett Birkhoff. Paper. Pages xiii+186. 12.5×20 cm. 1955. Dover Publications, Inc., 1780 Broadway, New York 19, N. Y. Price \$1.75, Cloth \$3.50.

SCIENCE IN YOUR LIFE FOR GRADE 4, by Herman and Nina Schneider. Cloth. 314 pages. 14.5×22 cm. 1955. D. C. Heath and Company, 285 Columbus Avenue, Boston 16, Mass. Price \$2.36.

SCIENCE IN OUR WORLD FOR GRADE 5, by Herman and Nina Schneider. Cloth. 314 pages. 14.5×22 cm. 1955. D. C. Heath and Company, 285 Columbus Avenue, Boston 16, Mass. Price \$2.28.

SCIENCE FOR TODAY AND TOMORROW FOR GRADE 6, by Herman and Nina Schneider. Cloth. 378 pages. 14.5×22 cm. 1955. D. C. Heath and Company, 285 Columbus Avenue, Boston 16, Mass. Price \$2.44.

NON-EUCLIDEAN GEOMETRY, by Roberto Bonola. Paper. Pages xii+268+xxx +71+50. 12.5×20.5 cm. 1955. Dover Publications, Inc., 1780 Broadway, New York 19, N. Y. Price \$1.90, Cloth \$3.95.

STATISTICS OF PUBLIC ELEMENTARY AND SECONDARY EDUCATION OF NEGROES IN THE SOUTHERN STATES: 1951-52, by Carol Joy Hobson, *Research Assistant, Research and Statistical Standards*. Circular No. 444, May 1955. Pages III+18. 20×26 cm. Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C. Price 20 cents.

SALARIES AND OTHER CHARACTERISTICS OF BEGINNING RURAL SCHOOL TEACHERS: 1953-54, by Wells Harrington and Mabel C. Rice, *Research and Statistical Standards Section*, under the General Direction of Herbert S. Conrad, *Chief, Research and Statistical Standards Section*. Circular No. 446, May 1955. Pages iii+16. 20×26 cm. Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C. Price 20 cents.

CAREGIE TECHNICAL PAPER. 72 pages. 22×29.5 cm. April, 1955. Carnegie Institute of Technology, Schenley Park, Pittsburgh 13, Pa.

BOOK REVIEWS

SCIENCE AWAKENING, by B. L. Van der Waerden. English Translation by Arnold Dresden. Cloth. 306 pages. 17.5×25 cm. 1954. P. Noordhoff, Ltd., Publishers, Groningen, Holland. Price \$5.00.

Those who are unable to read Dutch will welcome this English translation of *Ontwakende Wetenschap*. The original work was published in 1950 to provide an up-to-date history of ancient mathematics for the general reader. The research which had been done since the appearance of *The History of Greek Mathematics* by Sir Thomas Heath in 1921 had produced a real need for a new survey of the field. The greatest advances had come from three directions: (1) the translation and interpretation of many mathematical cuneiform texts from Babylonia, (2) a deeper study of the relationship between the writings of Plato and the fundamental concepts of Greek mathematics, and (3) a comparative analysis of various parts of Euclid's "Elements." Two of the most striking results of this research are the discoveries of the Babylonian origin of the Greek tradition and the remarkable mathematical talents of Theatetus. Even those who are thoroughly acquainted with the older accounts of Greek mathematics will find this new presentation absorbing.

The author is best known for his research and textbooks in modern mathematics but his scholarly work includes papers in the history of Greek and Babylonian science. His writing style is lively and enthusiastic. In *Science Awakening* his materials have been selected and organized with great care. In the short space of three hundred pages he has assembled a remarkably complete and stimulating introduction to the mathematical culture of ancient Egypt, Babylonia, and Greece. The absence of a bibliography somewhat mars the usefulness of this book as a reference work. Throughout the text there are references to source material and to more extended treatments of various topics but these would be more accessible if they were assembled in one place.

The arguments presented in *Science Awakening* do not make use of any mathematical facts beyond those of high school geometry and algebra. For the fullest appreciation of the chapter on the Alexandrian era, however, an acquaintance with analytic geometry and elementary calculus is advisable for two reasons. On the one hand, the mathematical techniques of Archimedes and Apollonius brought them to the threshold of these subjects and, on the other, the additional mathematical experience gained in these more advanced subjects makes reading the proofs much easier. The arguments are given with a certain amount of mod-

ernization so as to remove any unnecessary barriers to rapid understanding and yet the flavor of the original work is preserved.

ROBERT E. MacKENZIE
Indiana University
Bloomington, Indiana

DIFFERENTIAL AND INTEGRAL CALCULUS, by Harold Maile Bacon, Ph.D., *Professor of Mathematics, Stanford University*. Cloth. Pages vii+547. 16×23.5 cm. Second Edition. McGraw-Hill Book Co., Inc., 330 West 42nd St., New York 36, N. Y. 1955. Price \$6.00.

This is the second edition of a standard first year text. The first edition was reviewed in *SCHOOL SCIENCE AND MATHEMATICS*, Vol. 43, page 599 (June, 1943). This reviewer has always held the first edition in high regard; it seemed to come nearer to reproducing what an experienced teacher might say to his classes than almost any other text. Hence it is with regret that some features of the first edition have been dropped, for example, the section on applications of the concept of curvature and again, material on page 344 of the first edition which showed the equivalence, except for a constant, of two integration formulas. However, certain features which are retained continue to be outstandingly valuable. These would include examples on pages 86-87 which illustrate the fact that the mathematical maximum may not satisfy the conditions of the problem, and that there may be a maximum without the derivative being zero. In the same connection, the alternative method of attacking maximum and minimum problems (Section 38) where we require a relationship between variables, rather than an actual numerical value, is very well handled. On page 110 a footnote explains *why* we use the notation $\arcsin x$; on page 113 the author very wisely points out that we could select other branches of the curve as the principal branch, we merely make the choice which proves most convenient. In the discussion of integration by parts this reviewer likes the "alternative method" which is presented; likewise the author's explanation of why the arbitrary constant was not needed when finding v from dv .

Problem lists seem ample in length, in general, answers are provided for odd numbered examples. The inclusion of a few numerical tables would be appreciated by some instructors. This is a good text.

CECIL B. READ
University of Wichita

TRIGONOMETRY, by Roy Dubisch, *Associate Professor of Mathematics, Fresno State College*. Cloth. Pages xiv+396. 14.5×21 cm. The Ronald Press Company, New York, N. Y. 1955. Price \$5.00.

The author in his preface points to criticism of the traditional course in trigonometry, with its overemphasis on triangle solution and angles. Certainly this text is a distinct and refreshing innovation. In fact, this reviewer wonders whether it may not be so radical a departure from traditional texts that teachers may fear to attempt its use. At least the book merits careful consideration before it is rejected.

It may be well to consider first some of the unusual features. At least one which is immediately noted is the fact that for all practical purposes angles are not mentioned before Chapter 8 (page 153). But, how does one teach trigonometry without using angles? The trigonometric functions are featured as functions of real numbers, which necessitates an introductory treatment of the function concept—defined as a rule of correspondence between two sets of objects. A striking innovation is the use of a cardboard protractor, known as an "arc length protractor" and the introduction of the "arc length function" by which an ordered pair of real numbers are associated with any real number. By use of this function, the sine and cosine functions are defined. Since these are functions of *numbers*, we encounter "the double and half number identities" rather than double angle formulas. Similarly, we find that $\arcsin(x)$ means "a number whose

sine is x ." In Chapter 11 we find a complex number defined as an ordered pair of numbers (r, θ) . With the solution of triangles playing a minor role, it is not surprising to find the treatment of the law of tangents and the half angle formulas, as they relate to solving triangles, relegated to the appendix. Perhaps enough has been indicated to show in part where the emphasis is placed.

Whether or not one is pleased with the change in emphasis, as contrasted with that of the traditional text, one cannot help but be favorably impressed with certain features which are often weak spots in a text. There is no confusion in the definition of the mantissa of a logarithm; the treatment of significant figures seems sufficiently rigorous for the purpose of the text (it might have been pointed out on page 268 that the "even number" rule in rounding off is a purely arbitrary assumption); it is clearly pointed out that there is not general agreement as to the principal inverse functions, and exercises discuss alternative definitions. In the discussion of identities it is emphasized that there is a restriction that the variables are in the domain of the functions in question, thus avoiding undefined expressions. There are many problems taken from later mathematics, engineering, and scientific work; certain problems, indicated by an asterisk, are either of a more difficult nature, or make references to articles in journals or books. The better student who attempts any appreciable number of these will become familiar with at least a part of his school's mathematical library.

A few features, mostly of a minor nature, did not appeal to the reviewer. It seems there is an excessive number of footnotes; the type in the tables seems unduly small; taking $\pi=3$ along the x -axis (pages 83-84) produces only slight distortion, it is true, but one wonders if one loses more than is gained (at least the author points out what has been done, which is more than is often the case). On page 253 it is doubtful if a five place table justifies using angles to the nearest second. On page 25, Exercise 3 implies that in biblical times the value $\pi=3$ was used; a reading of the versus in question "... it was round ..." does not necessarily imply *circular*.

In most places the number of exercises seems ample; answers are provided to odd numbered exercises, and a separate pamphlet gives answers to even numbered exercises.

The teacher who wishes to emphasize triangle solution will find this text entirely unsuitable; the teacher who feels that too much of trigonometry has been numerical, rather than analytical, will wish to give this book very serious consideration. The student who has previously had some work in trigonometry will hardly recognize any similarity to what he has previously studied, at least for the first few chapters of this text. To this reviewer, the text seems a distinct improvement over the majority of texts now available, at least for the student who plans to continue his studies in mathematics.

CECIL B. READ

MATHEMATICS IN TYPE, by The William Byrd Press. Paper. Pages xii + 58. 16×24 cm. 1954. The William Byrd Press, Inc., 1407 Sherwood Ave., Richmond, Va. Price \$3.00. (Fifty per cent discount to educational institution staff members.)

As Mathematics Editor of SCHOOL SCIENCE AND MATHEMATICS, this reviewer has seen so many manuscripts submitted in a form almost entirely unsuitable for sending to the printer that he welcomes this booklet. The booklet is concerned primarily with material of somewhat advanced mathematics, and the problems involved from the printer's viewpoint. At the same time a large part of the material is also applicable to manuscripts covering mathematics of a more elementary nature.

The first portion of the book describes the methods of composing printed copy, and hence gives reasons why certain standards are almost essential from the point of view of economic printing of the material. The section of perhaps greatest interest to the one preparing the manuscript deals with setting and spacing. By comparing mathematical symbols to forms of speech (for example, +, -, and = are verbs and conjunctions; letter and figure symbols generally represent

nouns) there are obvious reasons for certain basic spacing rules. Another section, of perhaps equal importance, relates to preparing and marking the manuscript. The last portion of the book displays a list of available figures, alphabets, and symbols available without the necessity for special manufacturing.

Certainly this booklet should be in the hands of any one who is preparing any appreciable amount of mathematical copy for the printer. It would be a valuable addition to any university library. In many cases style form standards for thesis writing are very vague with reference to the preparation of mathematical material. Even though the particular thesis is not being prepared for the printer, the material in this booklet will give answers to questions not found in a manual prepared for general use.

CECIL B. READ

COLLEGE CHEMISTRY, Second Edition, by Linus Pauling, Professor of Chemistry, California Institute of Technology. Cloth. Pages xii+685. 15×23.5 cm. 1955. W. H. Freeman and Co., San Francisco, California. Price \$6.00.

This is a revision of a text that had previously been well received.

The first observable difference is a change in the general organization of the book; chapters have been grouped into natural units or parts. The groupings are as follows:

Part One: An introduction to chemistry.

Part Two: Some aspects of chemical theory.

Part Three: Some non-metallic elements and their compounds.

Part Four: Water, solutions, and chemical equilibrium.

Part Five: Metals, alloys and the compounds of metals.

Part Six: Organic chemistry, biochemistry and nuclear chemistry.

As in the first edition emphasis is given to the atomic and molecular structure of matter and the properties of substances are explained in terms of this structure. More space is given to the subject of organic chemistry. The arrangement is logical and explanations are clearly stated, the paper, print and illustrations are very good. The total number of pages were reduced from 715 to 697. There are over 200 well selected illustrations.

Any college instructor considering the adopting of a new text for beginning students in chemistry should give serious consideration to *College Chemistry* by Pauling.

GERALD OSBORN
Western Michigan College
Kalamazoo, Michigan

UNIVERSITY PHYSICS, Second Edition, by Francis Weston Sears, *Massachusetts Institute of Technology*, and Mark W. Zemansky, *The City College of New York*. Cloth. Pages viii+1031. 15×23 cm. 1955. Addison-Wesley Publishing Company, Inc., Cambridge 42, Mass. List Price \$10.00. College Adoption Price \$8.50.

During the past five years, Sears' and Zemansky's *University Physics* has become recognized as an outstanding textbook for students of science and engineering. The section on mechanics is particularly well developed and is lucid and accurate in interpretation of the fundamental concepts of physics. The treatment of electricity deserves special mention as well. It is rare that the fundamental ideas of electricity are presented on the elementary level in the proper, logical and fundamental way. In fact, no part of the book can be criticized for lack of accuracy or clarity. This is chiefly owing to the fact that the author's emphasis throughout is on the understanding of basic principles.

The second edition of this popular book is now available and contains a number of additions and revisions which probably have come as suggestions from numerous instructors. The outstanding qualities of the first edition remain with no changes. Some diagrams have been redrawn, numerous problems are worked out in the text, and a certain amount of new material has been added. Among

the new topics, the more important ones are: convection, surface tension and kinetic theory of gases (the latter being still somewhat brief). The discussion of Ampere's Law, mirror and lens formulas, and polarization have been extended.

An outstanding feature of the second edition, as of the first edition, is the set of problems. A completely new set of problems has been incorporated and the old set reprinted at the end of the book. The problems for the most part are original and challenging to a degree rarely found in the average text, and are most effective as a test of the students' understanding of the fundamentals.

A student interested in mastering the ideas of physics and who has the knowledge of a little calculus will not find a book more useful to this end.

ROGER WILKINSON
Indiana University
Bloomington, Indiana

MATHEMATICS—A THIRD COURSE, by Myron F. Rosskopf, *Associate Professor of Mathematics Education, Teachers College, Columbia University*, Harold D. Aten, *Oakland, California*, and William D. Reeve, *Professor Emeritus of Mathematics, Teachers College, Columbia University*. Cloth. Pages iii+438. 15×21.5 cm. 1955. McGraw-Hill Book Co., Inc., 330 West 42nd St., New York 36, N.Y. Price \$3.48.

This book covers three phases of mathematics—algebra, trigonometry, and analytic geometry. It gives a complete coverage of algebra, considerable amount of trigonometry and introduces very nicely analytic geometry. Advanced mathematics students will find a challenge in this text.

Good work is done in explaining the theory. Enough problems from simple to more complex are given. The chapter on approximations is very good and is well written.

There are a few places that could be explained in a better manner, and a few places which will challenge a good teacher for further explanation. For example, on page 191 it is stated, "24 10' is a four significant digit figure," but on page 268 this statement is found. "C 251.2. Since the angle is given to the nearest 10 minutes, C should have three significant digits." This might be confusing to a student, but herein lies the challenge for good teaching. In a few other places, statements were made, which taken within themselves, might be questionable. But, here again, a good teacher could take these points and go ahead with exceptions and challenge the class to more profound thinking.

This book, if taught correctly, would produce mathematics students who should not have trouble in taking college mathematics.

A. PRYCE NOE
University School
Bloomington, Indiana

ATOMS, TODAY AND TOMORROW, by Margaret O. Hyde. Eighty-three diagrams and line drawings by Clifford N. Geary. 144 pages. 14.0×21.0 cm. 1955. Whittlesey House, McGraw-Hill Book Company, Inc., 330 West 42nd Street, New York 36, N.Y. Price \$2.50.

This is the seventh book on "Atoms" to pass over this sampler's desk for review since 1948. It is, however, the only one of the seven beamed toward teen age readers.

Starting with the "Geiger counter prospector" this story of the "Atoms," that are energy producers, is followed to their discard as "reactor ashes." Bombs take a side-seat in the performance with the less spectacular, though possibly even more potent, peace-time services at the stage center.

The twelve chapter titles are: Prospecting with Geiger counters; What is atomic energy?; Atom smashing; The pile; Uranium "classified"; Radio-activity hazard; Doctor and atoms; Atomic farming; Atoms in overalls; Electricity from atoms; Atomic travel and Atoms in your future.

Stress is upon civic assets. This approach is initiated by reference to President Eisenhower's proposal of "cooperation in the peaceful use of atomic energy" to the United Nations Assembly and is faithfully followed in the chapters thereafter.

Mrs. Hyde carries her reader into the narrative by an informal, almost conversational, sharing of the experiences related. Whether listening to the click of the Geiger counter with the prospector or breathlessly watching as Dr. Fermi directs the cadmium rod withdrawal at the Chicago "squash court" pile the reader gets information and motivated experience by vicarious participation. This desirable reader response is facilitated by concise anecdotal narrative, expressed in short sentences, with sparse use of specialized vocabulary, packaged in short chapters and furthered by illuminative and informative diagrams and line drawings. Even though tempted by almost infinite magnitudes, the author has used commendable restraint in the number of superlatives permitted to parade through the exposition. Even so, the critical specialist will probably "lift his eyebrows" over the appearance of the adjectives: wonderful, famous, fascinating, amazing, mysterious and exciting. Let it be said the word "magic" did not get in.

While Mrs. Hyde is, in no sense, an authority in this field, her roster of consultants impresses one with confidence in her sources for the facts of her book. Her background of previous institutional connections also bolsters that estimate.

There is a "Glossary of atomic language" and an index. Both, however, are disappointingly brief. There is no book list to encourage further reading. That may be due to the fact that such books for teen agers are few. Since that may be the case all the more reason that this volume should be in every high school library and upon the desk of every forward-looking teacher.

B. CLIFFORD HENDRICKS
Longview, Washington

THOSE ATOMIC SECRETS DISCOVERED BY MOST NATIONS

In a half dozen fields the scientists of various nations meeting at the International Conference on Peaceful Uses of Atomic Energy have confirmed what most of them suspected:

Their hard-won secrets, jealously guarded, have been discovered by other nations through the brains and scientific sweat of their scientists. Specifically:

The ability of the fissionable or atomic power producing elements to capture neutrons, their cross-section, as determined in the United States, Britain, Russia, France and Norway agree so closely that differences can not be detected on a diagram. This fundamental information about plutonium, uranium 235 and 233, was closely guarded heretofore. Necessary for making bombs and reactors alike, the world has been given this information which each national group was hiding.

The separation of the metals zirconium and hafnium was reported by six processes in about as many nations, all unannounced heretofore. While the United States is the only country with sizable commercial production of zirconium, other localities will be able to produce this metal which is useful in coating the fuel in atomic reactors.

When medical experts of the USSR compared their radiation safety practices with those of the western block, the same figures for safe exposure of workers in atomic industry plants appeared, about 50 milliroentgens per day being safe. The Russians like to set their controls daily while the U. S. and United Kingdom prefer to use 300 milliroentgens a week as the figure after which the worker is given an enforced vacation for his own protection. There is no agreement on what lifetime dose is dangerous.

There was agreement there is danger of influencing unborn generations through genetic changes produced by very low levels of radiation, but the extent of such danger is not agreed upon by experts of different nations or even of the same nation.

PROPOSED AMENDMENTS TO THE BY-LAWS OF THE CENTRAL ASSOCIATION OF SCIENCE AND MATHEMATICS TEACHERS

The following proposed amendments to the By-Laws of the Central Association of Science and Mathematics Teachers will be brought before the membership for vote at its annual business meeting to be held in Detroit, Michigan, November, 1955. To assist the members in studying the proposed revisions, each section of the By-Laws under consideration is printed in full in the left column as it will read if the proposed amendment, appearing in italics, is voted. The *present* wording of the portion of the section to be amended appears in a parallel position at the right.

PROPOSED AMENDMENTS

PRESENT WORDING

PROPOSED ARTICLE I—SECTION III.

TERMINATION OF MEMBERSHIP:

The membership of any member of the Association shall be automatically terminated for any delinquency of dues, or other charges owing to the Association, continued for a period of six months. *Any member of the Association may be expelled for any reason deemed sufficient by the Board of Directors by a two-thirds affirmative vote of all those present at any meeting of the Board of Directors, or by a two-thirds affirmative vote of all the members present at any meeting of the Association.* But no vote of expulsion may be taken unless written notice shall have been given not less than thirty days prior to the date for said meeting to the member proposed to be expelled, which said notice shall advise said member of the time and place of said meeting and the reasons for which expulsion is proposed. Like notice shall be given to all other members of the Association. It shall be the privilege of the member proposed to be expelled to appear and be heard by the members at the meeting at which a vote is to be taken on the question of expulsion.

Any member of the Association may be expelled for any reason deemed sufficient by the Board of Directors by a two-thirds affirmative vote of all those present at any meeting of the Directors or the members of the Association.

PROPOSED ARTICLE II—SECTION I.

TIME OF MEETING: (a) ANNUAL MEETINGS:

The annual business meeting shall be held on the second day of the annual convention as set by the Board of Directors.

The annual meetings shall be held on the second day after the last Thursday in November of each year.

PROPOSED ARTICLE III—SECTION I.

OFFICERS:

The officers of this Association shall be a

President, a Vice-President, a Secretary, a Treasurer and Business Manager, an Editor of the Journal, and an Historian. *One or more Assistant Secretaries, Assistant Editors, and Assistant Treasurers may be appointed by the Board of Directors.*

PROPOSED ARTICLE III—SECTION III.

ELECTION, TENURE OF OFFICE, COMPENSATION:

The President and Vice-President shall be elected by the members of the Association at the annual meeting and shall serve for a term of one year or until their successors are elected. *The Treasurer and Business Manager, Editor of the Journal, and Secretary-Historian shall be appointed for a term of three years by the Board of Directors at a meeting to be held following the annual meeting of the Association, or at the Spring meeting of the Board of Directors. The Secretary-Historian shall take office immediately following appointment. The Treasurer and Business Manager, and Editor of the Journal shall take office at the beginning of the fiscal year following their appointment. They may be reappointed.* The compensation of the officers, if any, shall be fixed by the Board of Directors.

PROPOSED ARTICLE III—SECTION IV.

(d) TREASURER AND BUSINESS MANAGER:

The Treasurer and Business Manager shall collect all dues and hold all moneys and keep a record of all receipts and disbursements. He shall give a report at the annual meeting of the Association. He shall pay out funds on the order of the Board of Directors and the Executive Committee.

(Amend to delete the final sentence appearing at the right.)

One or more Assistant Secretaries and Assistant Treasurers may be appointed by the President.

The Treasurer and Business Manager, Editor of the Journal, Historian, and Secretary shall be appointed by the Board of Directors at a meeting to be held following the annual meeting of the Association, and shall serve for a term of three years. Their terms may be renewable.

He shall also act as Business Manager of the Journal.

ISAAK WALTONS NEED BULBS IN BAIT BOX

Fishermen will be taking along electric light bulbs for bait soon.

Norwegian technicians have devised a metal container with four windows through which an electric light shines. Lowered into the water, the light attracts fish to swim close to the apparatus.

The lower part of the container, lying in darkness, is equipped with hooks. When the gear is jerked upward, the dazzled fish are caught, theoretically anyway.

This is one of several uses for the "light ray fishing sinker" says the U. S. Fish and Wildlife Service, reviewing a report of the apparatus from the *Norwegian Fishing News*.

In trawling, several of the light ray sinkers are placed on a special hoop set at the mouth of the trawl net. The fish, attracted to the light, gather in front of the net, thus considerably increasing the catch.

DR. POTTER AND ASSOCIATES MAP THE CELL

University of Wisconsin scientists are making a three dimensional map of the interior of a human body cell.

Dr. Van R. Potter, speaking at the 128th annual meeting of the American Chemical Society in Minneapolis, told how this mapping furthers the understanding of the mechanisms involved in the production of nucleic acids—end products of the metabolic processes by which body cells transform food into energy and new cells.

It also sheds light on where some of the metabolic products go, charting the progress of incoming molecules along the pathways they take and thus providing cell chemists with a better geographical orientation, he explained.

The understanding of metabolism and cell growth is basic research toward learning what makes cells run rampant in degenerative diseases like cancer.

In the past two years Dr. Potter and his associates, Dr. Liselotte Hecht and Dr. Edward Herbert, have duplicated in cell-free systems (those which contain no intact whole cells) some of the chemical events that have been shown to occur in cells of living animals during the production of nucleic acid.

Dr. Potter, who is professor of oncology at UW, described experiments in which cells were broken open and their contents separated into components. The solid particles included the nucleus, responsible for the precise mechanism of cell-division; the mitochondria, large thread-like granular particles, which generate the main power supply of the cell; and the microsomes, small bodies which now appear to be a center of protein and nucleic acid formation.

Dr. Potter said that it seems likely that the nucleus is a second center of protein and nucleic acid formation and that in some respects the two centers are competitive while in other respects they are cooperative in action.

The Wisconsin scientist told how molecules like orotic acid, known to be used in forming parts of the nucleic acid products, were labelled with radioactive carbon atoms and added to various combinations of cell parts. It was shown that none of the cell parts could incorporate the acid alone, but various combinations of cell parts could do the job together.

SCIENTISTS STUDY SUBSTANCE 300 TIMES SWEET AS SUGAR

Sweethearts of the next generation may be telling each other they are "sweet as stevioside." For that crystalline chemical from the leaves of a wild Paraguayan shrub has been found to be 300 times sweeter than the usual standard of lovers, table sugar.

In the meantime, however, unromantic scientists of the National Institute of Arthritis and Metabolic Diseases are subjecting this super-sweet compound to a series of tests to learn its chemical structure and to discover any practical applications for it.

Dr. Hewitt G. Fletcher Jr., of the National Institutes of Health, reports that stevioside is made up of very large molecules containing only three ingredients, carbon, hydrogen and oxygen. Included in each large molecule are three sub-molecules of glucose, the common cane sugar.

Stevioside does not have the bitter after-taste of saccharin and it apparently causes no ill effects on experimental animals who receive it as food. For the diet-conscious, stevioside seems to have little or no food value.

Source of stevioside is a small shrub, *Stevia rebaudiana*, that grows wild in Paraguay and a few near-by areas in Argentina and Brazil. It was seriously considered as a sugar substitute in England during World War II. Difficulty in cultivating the plant and cheaper costs of saccharin, however, have kept stevioside a mere curiosity up to now.

Dr. Fletcher holds out hopes that stevioside may find eventual use in medicinals or in their synthesis. His report appeared in the journal, *Chemurgic Digest* (July-Aug.).

WORLD POPULATION BOOMS

The world's population is booming. Since the beginning of World War II it has grown by almost one-third billion and now exceeds two and one-half billion. Put it down to more babies and fewer deaths.

A new record low death rate has been set by American wage earners and their families. For the first six months of 1955 it was the lowest in the nation's history.

Even for heart, blood vessel and kidney disorders, the death rate among the wage-earner group was below last year's, 344.9 per 100,000 compared with 347.1.

These birth and death figures come from Metropolitan Life Insurance Company statisticians. For their millions of industrial policyholders the death rate for the first six months of 1955 was 644.4 per 100,000 insured compared with the previous low of 652.1 set last year.

The death toll from cancers in the insured group stayed at last year's level, about 129 per 100,000.

Birth rates in Europe, Japan and the Philippines have fallen to or below pre-World War II levels except for France and Norway. In the United States and other English-speaking countries outside Europe, notably Canada, Australia and New Zealand, the birth rates have stayed a third or more above the prewar levels.

Less well-developed areas of the world continue to have high birth rates.

In the United States the margin of births over deaths has given a population increase at a rate of about one and one-half percent a year. The rate of increase has been even greater in many countries in Asia and Latin America. These countries, the statisticians point out, have potentialities of rapid population growth for many years to come.

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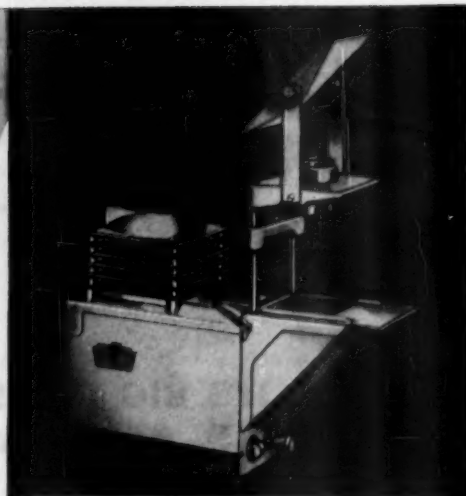
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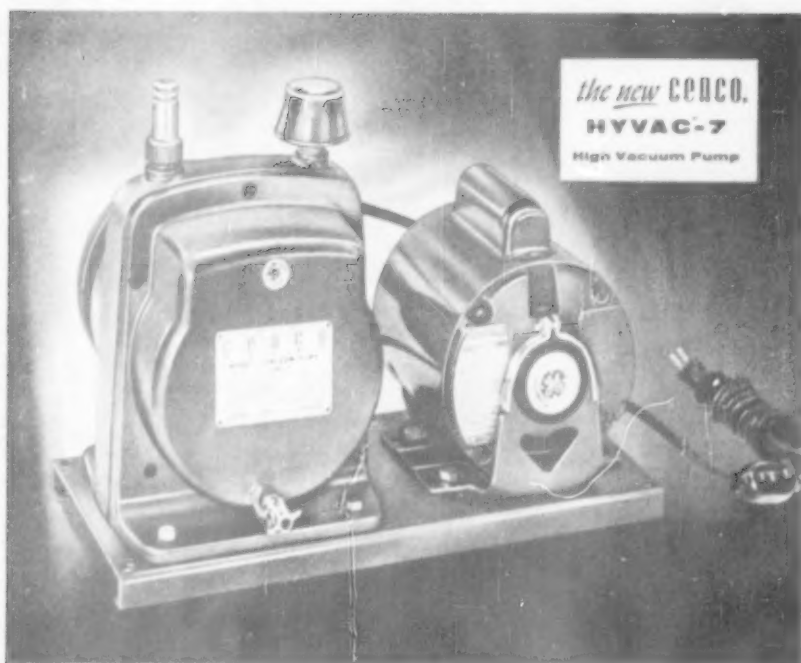
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